Supplemental material

Diffusion dynamics on multiplex networks

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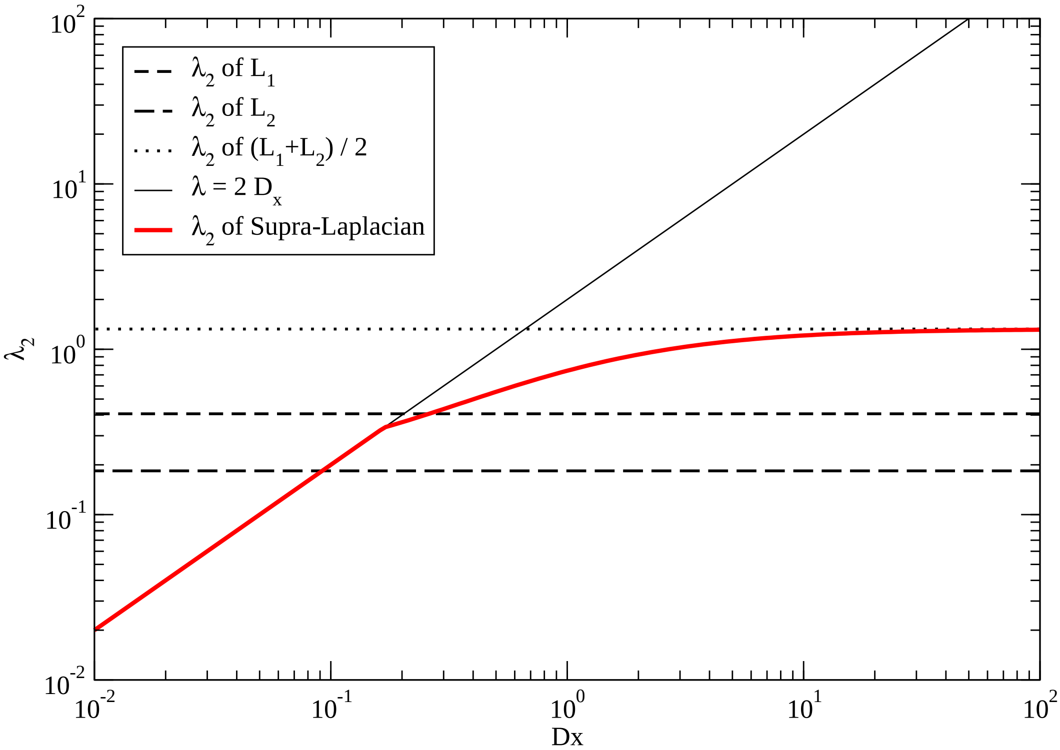

Figure 1: Comparison between the second smallest eigenvalues λ_2 of the different laplacians for a multiplex network consisting of two layers with 1000 nodes in each layer. The first contains a scale-free network with degree distribution $P(k) \sim k^{-2.5}$, and the second layer a scale-free network with degree distribution $P(k) \sim k^{-3}$.

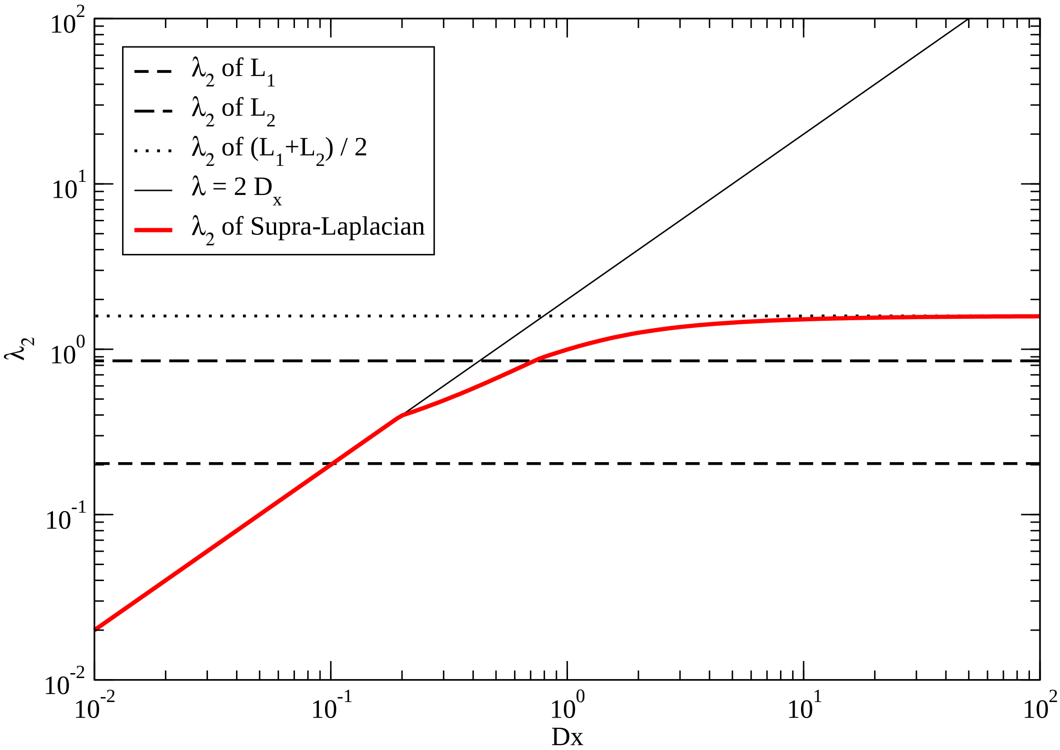

Figure 2: Comparison between the second smallest eigenvalues λ_2 of the different laplacians for a multiplex network consisting of two layers with 1000 nodes in each layer. The first layer contains a scale-free network with degree distribution $P(k) \sim k^{-2.5}$, and the second layer contains a random Erdös-Rényi network with average degree $\langle k \rangle = 8$.

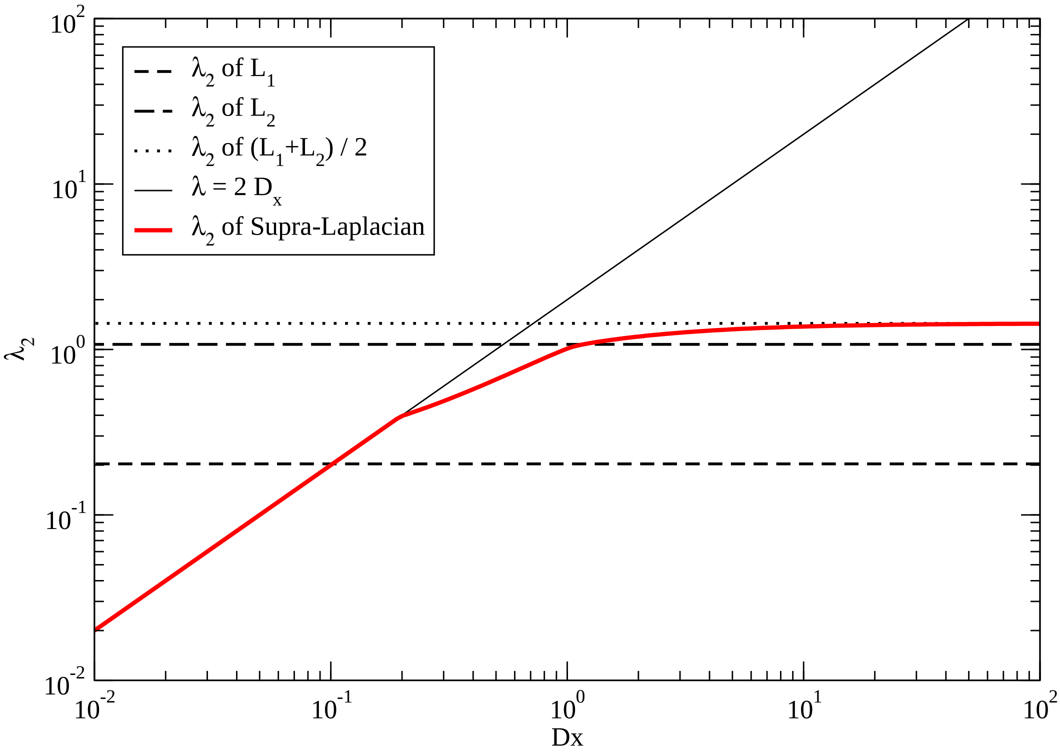

Figure 3: Comparison between the second smallest eigenvalues λ_2 of the different laplacians for a multiplex network consisting of two layers with 1000 nodes in each layer. The first contains a scale-free network with degree distribution $P(k) \sim k^{-2.5}$, and the second layer a small-world network with average degree $\langle k \rangle = 8$ and a replacement probability $r = 0.3$.

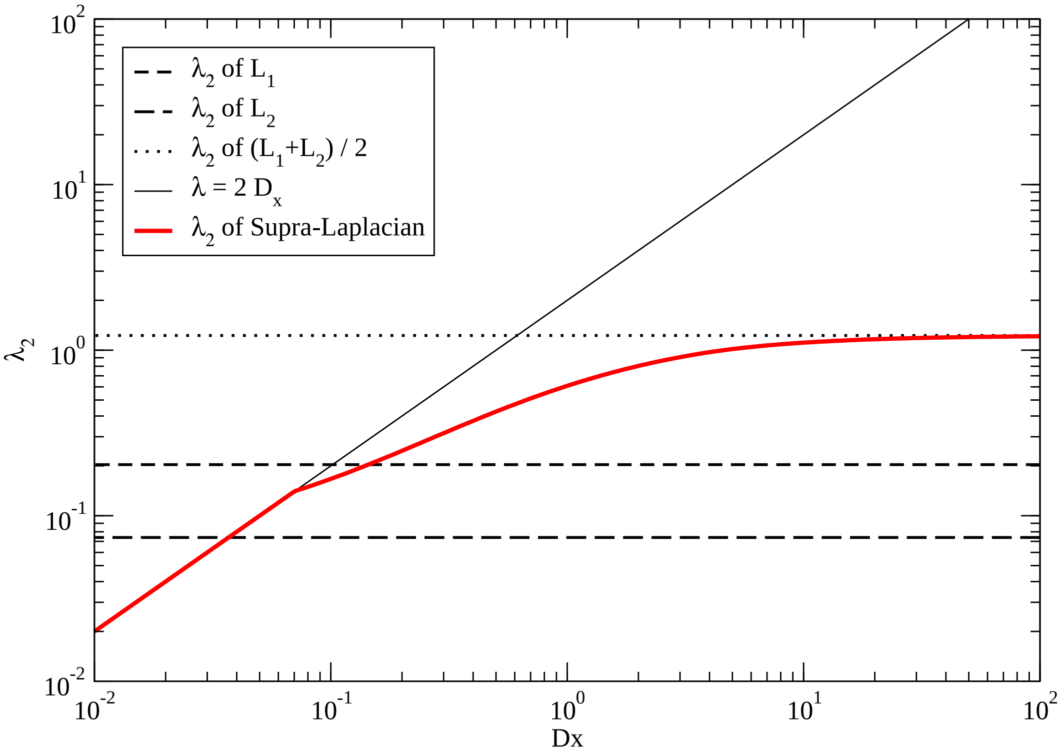

Figure 4: Comparison between the second smallest eigenvalues λ_2 of the different laplacians for a multiplex network consisting of two layers with 1000 nodes in each layer. The first contains a scale-free network with degree distribution $P(k) \sim k^{-2.5}$, and the second layer a 40×25 lattice with eight neighbors per node an periodic boundary conditions.

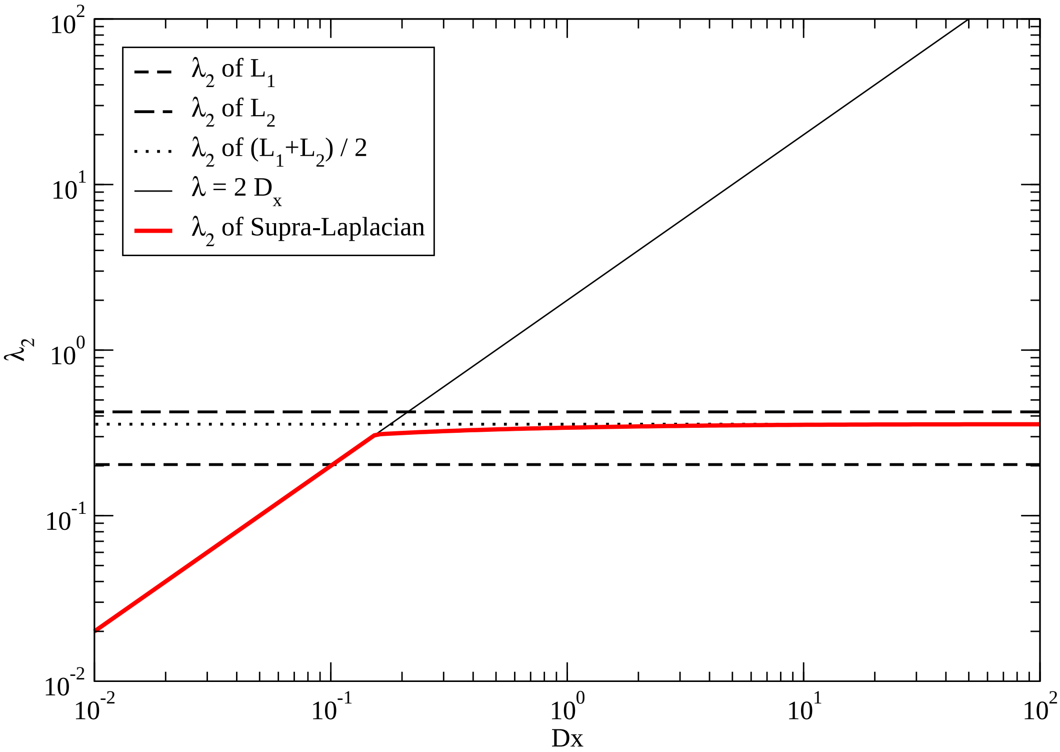

Figure 5: Comparison between the second smallest eigenvalues λ_2 of the different laplacians for a multiplex network consisting of two layers with 1000 nodes in each layer. The first contains a scale-free network with degree distribution $P(k) \sim k^{-2.5}$, and the second layer has been obtained from a copy of the first layer network with 400 extra random links. Here we observe the absence of super-diffusion. This is a consequence of the semi-superposition $(W_1 + W_2)/2$ being a (weighted) spanning graph of the network in the second layer W_2, thus according to Corollary 3.4 in [20] we have $\lambda_2((L_1 + L_2)/2) \leq \lambda_2(L_2)$.

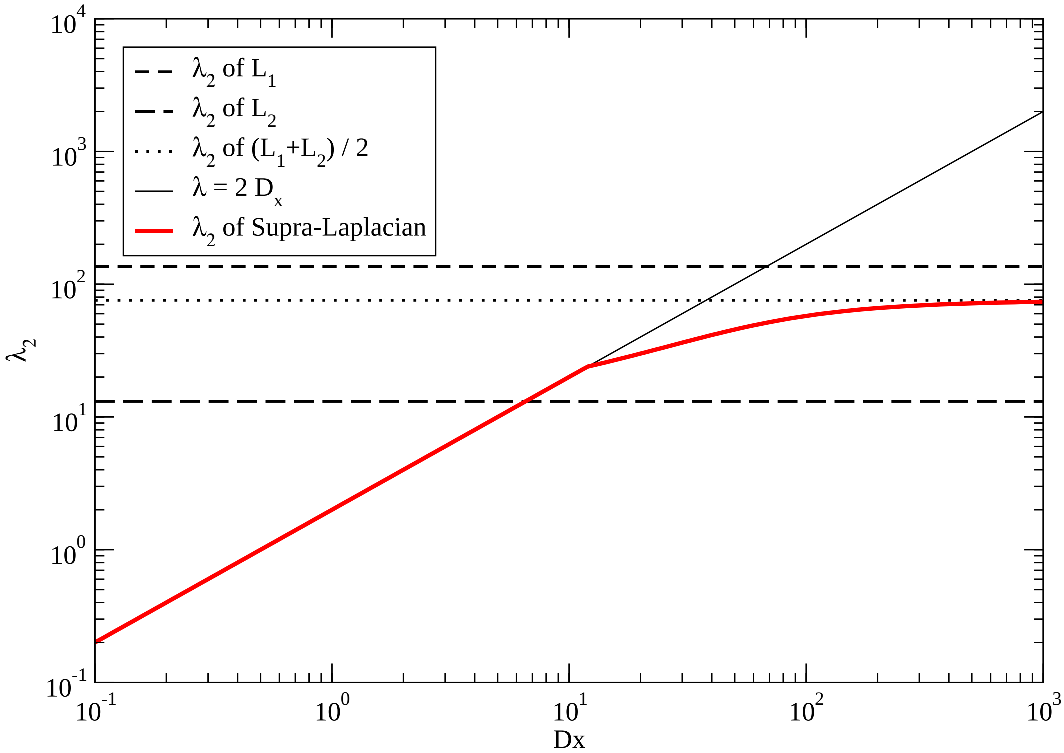

Figure 6: Comparison between the second smallest eigenvalues λ_2 of the different laplacians for a multiplex network consisting of two layers with 1000 nodes in each layer. The first contains a network structured in 4 communities of 250 nodes each, with average internal and external degrees $\langle k^{\text{int}} \rangle = 105$ and $\langle k^{\text{ext}} \rangle = 105$ respectively, and the second layer is similar but with $\langle k^{\text{int}} \rangle = 200$ and $\langle k^{\text{ext}} \rangle = 10$. The average clustering coefficients at each layer are 0.2336 and 0.7307 respectively, and the communities in both layers match.

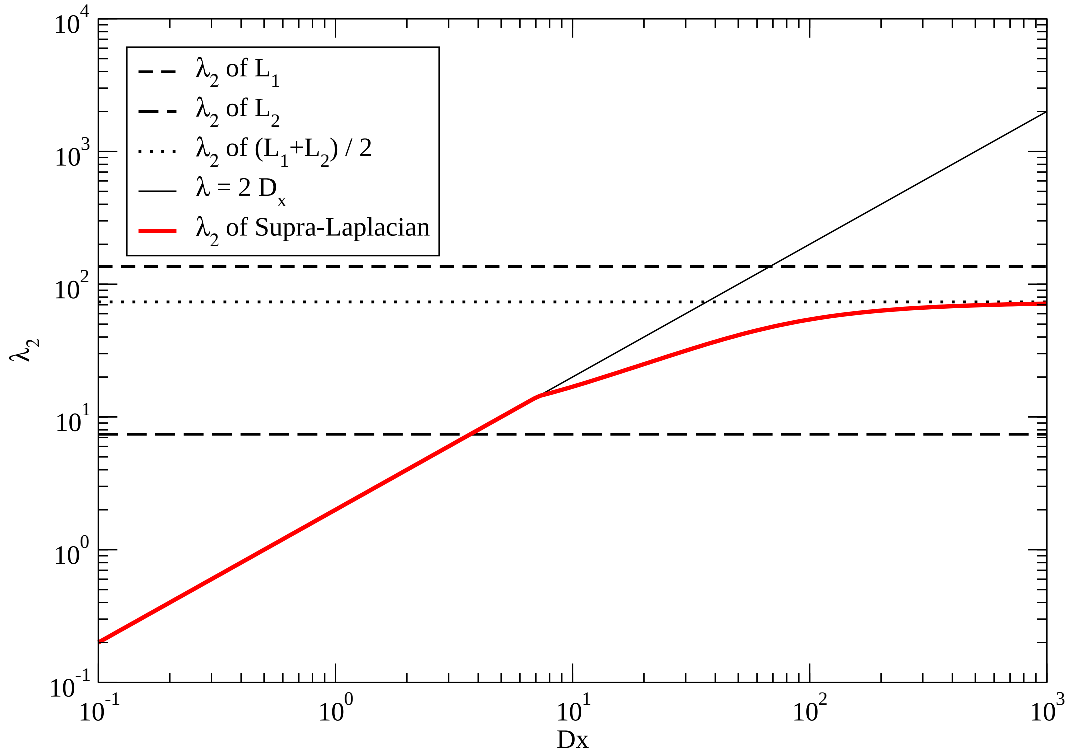

Figure 7: Comparison between the second smallest eigenvalues λ_2 of the different laplacians for a multiplex network consisting of two layers with 1000 nodes in each layer. The first contains a network structured in 4 communities of 250 nodes each, with average internal and external degrees $\langle k^{\text{int}} \rangle = 105$ and $\langle k^{\text{ext}} \rangle = 105$ respectively, and the second layer is similar but with 17 communities, $\langle k^{\text{int}} \rangle = 50$ and $\langle k^{\text{ext}} \rangle = 8$. The average clustering coefficients at each layer are 0.2336 and 0.6541 respectively.