

Universitat Rovira i Virgili

Escola Tècnica Superior d'Enginyeria



MASTER THESIS

Considering hierarchical structure of criteria in ELECTRE decision aiding methods

Olanrewaju Joseph Soniran Shofade

⟨olanrewajujoseph.soniran@estudiants.urv.cat⟩

Directors:

Dr. Aida Valls Mateu ⟨Department of Computer Engineering and Mathematics,
University of Rovira and Virgili⟩

Prof. Roman Slowinski ⟨ Institute of Computing Science, Poznan University of
Technology, Poznan, Poland⟩

Tarragona, June 2011

Acknowledgements

After the design of the models, the implementation and writing the documentation for the thesis, I have found out that the hardest thing for me to write is the acknowledgements because it is when I truly get to express how grateful I am to be surrounded by such great colleagues and a family that truly makes my life a joy.

I would firstly wish to acknowledge the support of the project (CTM2007-64490/TECNO), financed by the Spanish Ministry of Environment and the CENIT SOSTAQUA project for providing the medium through which the tests have been conducted.

In particular, I want to thank both my supervisors, Dr. Aida Valls and Prof. Roman Slowinski for all their support, technical input, fruitful comments and continuous motivation during the development phases of the thesis of which I have gained a lot of experience.

At the risk of overlooking some of my colleagues, I would like to dedicate these few lines to all the people I have worked with within the Itaka group especially Jordi Canals whose inputs have been particularly valuable.

Finally, to my mum for all her support, care and understanding throughout this period and to my amazing wife, to whom I owe a huge debt of gratitude, for the endless support, patience, compassion, understanding and encouragement. I am so blessed to be going through life with you and to have you as the mother of our precious Ivy the most wonderful little thing I could ever wish for.

Contents

Acknowledgements.....	ii
------------------------------	-----------

Abstract.....	xi
----------------------	-----------

1. Introduction	1
------------------------------	----------

1.1. Aims of the Master Thesis.....	3
-------------------------------------	---

1.2. Structure of the thesis	4
------------------------------------	---

2. State of the art.....	7
---------------------------------	----------

2.1. Basic concepts of multiple criteria decision aid.....	8
--	---

2.2. Preference Modelling	10
---------------------------------	----

2.3. Outranking methods	11
-------------------------------	----

2.3.1. Outranking Concept.....	13
--------------------------------	----

2.4. ELECTRE methods.....	14
---------------------------	----

2.4.1. ELECTRE I.....	16
-----------------------	----

2.4.1.1. Building of the Outranking Relation.....	16
---	----

2.4.1.2. Exploitation of the outranking relation	17
--	----

2.4.2. ELECTRE II [44]	18
------------------------------	----

2.4.3.	ELECTRE III [46]	19
2.4.3.1.	Building of the Outranking Relation.....	20
2.4.3.2.	Exploitation of the outranking relation	22
2.4.4.	ELECTRE IV [47].....	23
2.4.5.	ELECTRE TRI [48].....	25
2.4.6.	Concordance and Discordance relations in MCDA	26

3. ELECTRE III-H proposals.....33

3.1.	ELECTRE III-H methods	34
3.1.1.	ELECTRE III-H with median preorder: Version1	35
3.1.1.1.	Building the Outranking relation at the lowest level.....	36
3.1.1.2.	Exploitation of the outranking relation at the lowest level.....	37
3.1.1.3.	Building and Exploiting the Outranking relation at upper levels	39
3.1.1.4.	Algorithm for ELECTRE III-H with median preorder.....	42
3.1.2.	ELECTRE III-H by Net Flow Score ranking method: Version2.....	43
3.1.2.1.	Building the Outranking relation at the lowest level.....	43
3.1.2.2.	Exploiting lowest level outranking relation using the Net Flow Score procedure (NFS)	44
3.1.2.3.	Building the Outranking relation at upper levels	45
3.1.2.4.	Algorithm for ELECTRE III-H with Net Flow Score method	47
3.1.3.	ELECTRE III-H by Credibility Propagation (CRED) method: Version3	48

3.1.3.1.	Building the Outranking relation at the lowest level of the hierarchy	48
3.1.3.2.	Building the Degree of Credibility at upper levels of the hierarchy	49
3.1.3.3	Algorithm for CRED method	53
4.	Application of ELECTRE III-H methods to Environment sewage sludge disposal	55
4.1.	Problem Specification.....	56
4.2.	Criteria Hierarchy	59
4.3.	Data	61
5.	Tests and Results	63
5.1.	Application of version 1: ELECTRE III-H with median preorder	64
5.1.1.	Ranking Human Health Risk criterion at the lowest level of the hierarchy	64
5.1.2.	Ranking Human Health Risk criterion at upper levels	65
5.1.3.	Ranking Environment criterion at the lowest level of the hierarchy	67
5.1.4.	Ranking Environment criterion at upper levels of the hierarchy	69
5.1.5.	Final Ranking at root level of the hierarchy	70
5.2.	Application of version 2: Net Flow Score Method (NFS).....	72
5.2.1.	Ranking Human Health Risk criterion at the lowest level of the hierarchy	72
5.2.2.	Ranking Human Health Risk criterion at upper levels	75
5.2.3.	Ranking Environment criterion at lower levels.....	78
5.2.4.	Ranking Environment criterion at upper levels.....	80

5.2.5.	Final evaluation at root node of the hierarchy.....	80
5.3.	Application of version3: Credibility propagation (CRED) method.....	81
5.3.1.	CRED method at the lowest level	82
5.3.2.	CRED method application at upper levels	83
5.5.	Summary of results of the three methods.....	85
5.6.	Sensitive analysis.....	86
5.6.1.	Sensitivity with respect to the thresholds	87
5.6.2.	Results of sensitivity analysis	88
5.6.2.1.	Sensitivity analysis for version1	89
5.6.2.2.	Sensitivity analysis for version2	90
5.6.2.3.	Sensitivity analysis for version3	91
6.	Conclusions and future work.....	93
	Bibliography	97
	Annex.....	103

List of Figures

Figure 1 Concordance evaluated on criterion gj for ordered pair (a,b)	21
Figure 2 Valued outranking indices in ELECTRE III	27
Figure 3 The discordance index in the ELECTRE III method	28
Figure 4 Example of a flat structure of criteria.....	33
Figure 5 Example of a hierarchical structure of criteria	33
Figure 6 Hierarchical structure of criteria for sewage sludge disposals	60
Figure 7 Ranking lowest level Branch of Human Health Risk criterion.....	65
Figure 8 Human Health Risk criterion first upper level ranking.....	66
Figure 9 Human Health Risk criterion second upper level ranking	66
Figure 10 Human Health Risk criterion ranking	67
Figure 11 Ranking lowest level branch of Soil criterion in the Environment criterion branch	68
Figure 12 Ranking lowest level branch of Groundwater criterion in the Environment criterion branch	69
Figure 13 Ranking Soil criterion at first upper level	69
Figure 14 Environment criterion Ranking at highest level.....	70

Figure 15 Final evaluation at root level	71
Figure 16 Final graph, distillations and median preorder at root node	84
Figure 17 Plot of final ranking	86
Figure 18 Sensitivity analysis for ELECTRE III - H with median preorder: Version1	89
Figure 19 Sensitivity analysis for NFS method	90
Figure 20 Sensitivity analysis for CRED method.....	91

List of Tables

Table 1 Computation of the NCD-based MCC generalized framework	32
Table 2 Definition of criteria	57
Table 3 Table of Alternatives obtained with expert system.....	58
Table 4 Performance table for sewage sludge disposals	61
Table 5 Thresholds for sewage sludge disposals	63
Table 6 Concordance table for terminal criteria leading to population criterion	73
Table 7 Outranking matrix for the generation of population criterion	74
Table 8 Net flow score binary matrix	74
Table 9 NFS ranking for population criterion at the lowest level	76
Table 10 Performance table and rank scores for population risk criterion.....	76
Table 11 Performance table and rank score for ingestion dose.....	77
Table 12 Human health risk criterion performance table.....	78
Table 13 Soil performance table and rank scores for environment criterion branch.....	79
Table 14 Groundwater performance table and rank scores.....	79
Table 15 Performance table and rank scores for environment criterion	80
Table 16 Final ranking with the NFS method.....	81
Table 17 Credibility matrix inherited by population criterion	82
Table 18 Credibility matrix inherited by landscape criterion	82

Table 19 Final ranking produced by the three methods	85
Table 20 Original values of terminal criteria used in the test	87
Table 21 Original values of intermediate level criteria used in the test	87
Table 22 Values for first sensitivity test	88
Table 23 Values for second sensitivity test	88

Abstract

ELECTRE methods enjoy a wide acceptance in solving multiple criteria decision making (MCDM) problems. However, their application requires the definition of the criteria on a common level, such that, decision makers faced with problems containing hierarchical structure of criteria cannot directly apply the ELECTRE methods without transforming the hierarchy into a consistent family of criteria at the same level.

In most real-world applications, criteria are naturally defined in a hierarchical structure, distinguishing different levels of generality that model the implicit taxonomical relations between the criteria. In this thesis we propose three adaptations of ELECTRE methods that would accept hierarchical structure of criteria, without the need of the transformation to a common level. Using the structure of ELECTRE III method as a base for these adaptations, we designed the extended versions for multiple criteria ranking problems where recommendations are produced in the form of a partial order in the set of alternatives. This means that some alternatives may have ordered rank while others may be qualified as incomparable.

A crucial point of the work is the extension of the concept of concordance and non-discordance tests which permits the comparison of partially ordered alternatives by criteria at a given level. This way, a valued outranking relation is constructed at the corresponding level of the hierarchy. In the first version called *ELECTRE III-H with median preorder*, we exploit the outranking relation with the usual ELECTRE III distillation process to generate partial order of alternatives at each level. These partial orders are propagated through higher levels up to the root node where a final ranking is produced. The second version called the *ELECTRE III-H with Net flow score (NFS)* is similar to the first version with the exception that the exploitation of the outranking relation is completed with the net flow score procedure instead of the distillation process with median preorder. The third version called the *CRED* method directly propagate the outranking relation from lower levels of the hierarchy to higher levels, taking the non-compensatory effect of the weights of criteria into consideration. In this case, the ranking takes place solely at the highest level of the hierarchy. With these extensions, it will be possible to decompose a multiple criteria ranking problem with hierarchical criteria into sub-problems corresponding to subsets of criteria having the same upper level root, and to aggregate the results of these sub-problems up the hierarchy tree, so as to get a final partial order of the alternatives.

1. Introduction

The continuous economic growth and technological advancement over the last decades has led humans to face daily complex decision making problems in all fields, including, engineering, science, government enterprises, and in the business world. Problems such as urbanization, industrialization, increase in demand for water and energy supply, environmental pollution, shortage of natural resources, food, and many other challenges has to be dealt with. The frequency at which these decision making problems occur and the complexity of the problems require the development of a multidisciplinary approach with diverse mechanism, capable of analyzing and providing solution to problems with multiple criteria.

Multiple criteria decision aiding (MCDA) is one of the most widely used scientific methodologies of decision support that intends to improve the quality of decisions by helping decision makers to make rational decisions concordant with their preferences. Some of the most widely used MCDA methods are Analytic Hierarchical Process (AHP), Disaggregation/Aggregation Approaches (UTA*, UTAII, UTADIS, GRIP), MACBETH, PROMETHEE, Multi-Attribute Utility Theory (MAUT) and the ELECTRE methods. The ELECTRE methods are considered in this thesis for the purpose of adapting them to multiple criteria decision problems with a hierarchical structure.

The acronym ELECTRE stands for: ELimination Et Choix Traduisant la Réalité (Elimination and Choice Translating Reality). It is a well-known outranking decision aid methodology which helps a decision maker in either choosing a subset of best alternatives from a given set of alternatives, or in ranking the alternatives from the best to the worst, or in sorting the alternatives to some pre-defined and preference-ordered classes, based on evaluation of the alternatives on a consistent family of criteria. A characteristic feature of ELECTRE is the use of an outranking relation for the representation of decision maker's preferences. The underlying idea is that, if strong mathematical hypothesis which demands complex answers from decision makers, can be avoided, a better but less rich result (outranking relation) can be obtained by systematically comparing the alternatives on each criterion [1-3].

Since the development of the ELECTRE method by Bernard Roy in the mid 60's, several versions of the method have been proposed, starting from ELECTRE I and Is, for the selection of alternatives in a multiple criteria choice problems, ELECTRE II, for constructing a ranking of

alternatives using true criteria (without thresholds), ELECTRE III and IV, also used for constructing ranking of alternatives but differs from ELECTRE by its application of pseudo criteria (indifference and preference threshold), and ELECTRE TRI, is designed to solve sorting problems.

The ELECTRE methodology comprises two main procedures: the first part consists of the construction of one or several outranking relation(s) with the aim of comparing each pair of alternatives in a comprehensive way, followed by the exploitation procedure which is used for elaborating recommendation based on the analysis of the result of the outranking relations obtained in the first phase, for the purpose of solving a given decision problem. The outranking relation used for the construction of the first part of the ELECTRE methods is a reflexive and non-transitive binary relation, denoted by aSb , where “ a ” and “ b ” are alternatives and aSb implies that “alternative a is at least as good as alternatives b ”.

As to construction of the outranking relation, to justify that the hypothesis aSb is true for an ordered pair of alternatives (a,b) , two tests are performed: concordance test (checking if the coalition of criteria in favor of the hypothesis aSb is strong enough), and non-discordance test (checking if, among the criteria opposing the hypothesis aSb , no criterion in which alternatives b would be better than alternatives a by more than a veto threshold, would annul the hypothesis aSb). At the exploitation stage, different approaches are used depending on the nature of the problem. When the problem faced by the decision maker is a choice problem, a kernel of the outranking graph is searched for, alternatively, a special distillation procedure is applied on the valued outranking relation to produce a partial order on the set of alternatives when the problem is a ranking problem, or rather, optimistic and pessimistic procedures are used on the valued outranking relation to assign alternatives to preference-ordered classes for solving sorting problem.

ELECTRE methods have been widely considered as an effective and efficient decision aid method with successful applications in areas such as Agriculture and Forest Management[4], Energy sector [5], Environmental and Water Management [6], Finance [7], Military [8], Project Selection [9] and Transportation [10].

ELECTRE methods require that all criteria are defined on a common level and do not accept a hierarchy of criteria. Thus, to apply ELECTRE methods in case of a hierarchical structure of criteria, one needs to transform the hierarchy into a consistent family of criteria considered at the same level. The idea of transforming the criteria from hierarchical structure into flat structure may not be sufficient enough to provide the necessary and required solution to

decision problems where multiple decision makers are involved. For example, big companies where final decisions are taken at a very high level and the evaluation of the criteria is delegated to several services at lower levels of the hierarchy, require a decision making tool that can evaluate the problem at the lower level of the hierarchy and deliver results that will in turn be returned to the upper level for further integration until the “root” project is reached [11].

In most real-world applications, criteria are naturally defined in a hierarchical structure, distinguishing different levels of generality that model the implicit taxonomical relations between the criteria. The organization of criteria into hierarchies helps to acquire detailed knowledge of complex reality: the criteria are structured into its constituent parts, and these in turn into their own constituent parts, proceeding down the hierarchy to as many levels as the problem permits. Each step is focused on the understanding of a single component, while other components at this and all other levels are temporarily disregarded. Going up through the hierarchy, the global understanding of the complex problem is increased and a better picture of the problem can be obtained as a whole. More so, the knowledge of the different aspects of criteria can help “pace” or “leverage” various levels of criteria in order to overcome conflicts and achieve desired outcomes more effectively.

1.1. Aims of the Master Thesis

The aim of this Master Thesis is to propose an appropriate adaptation of the ELECTRE methods, which would accept a hierarchical structure of criteria, without the transformation of the criteria to a common level. Of the different versions of ELECTRE methods designed over the past years, the ELECTRE III method has been chosen as a basic method used for this adaptation. Originally, the ELECTRE III method is designed for a multiple criteria ranking problem, where all the criteria adopt a common level, in cases where problem contains hierarchical structure of the criteria, they are transformed to a single common level before quantify the relative importance of the criteria.

The main goal of this Thesis can be divided into the following categories:

- Study the ELECTRE methodology, in particular, ELECTRE III method.
- Extend the concepts of concordance and non-discordance tests to criteria with ordinal scales and partial orders of alternatives evaluated by these criteria.

- Propose an adaptation of ELECTRE III method based on the above extension, to deal with hierarchical structure of criteria (denoted ELECTRE-III-H).
- Implement the new ELECTRE III-H method.
- Apply the ELECTRE III-H method to the problem of management of the disposal of sewage sludge generated during the water cleaning process in wastewater treatment plants. The government encourages the reuse of sludge in order to achieve a sustainable water cycle. In the SOSTAQUA research project, a multi-disciplinary team considered all relevant factors to decide the best destination of sewage sludge from each plant. This is a multiple criteria ranking problem with a set of criteria organized into a hierarchical structure concerning three main aspects: economical costs, impact for humans, and impact on the environment and ecosystems [2, 12].

1.2. Structure of the thesis

The focus of this thesis is centered on the adaptation of the ELECTRE decision aiding methods to operate with criteria which has hierarchical structure. Section 1 of this work provides a description of the project and presents the fundamental notions of decision making and the relevance of MCDA for solving complex decision problems, highlighting some of the most widely used MCDA methods and their areas of application.

Section 2 provides a brief background and explanation of basic notations and definitions of MCDA. Firstly, a study of the important concepts relating to multiple criteria decision aid are discussed and secondly, a review of outranking method used in MCDA is presented with reference to different outranking methods which have been progressively developed and modified for decision making in real world application. The section concludes with a review of the different versions of the ELECTRE outranking methods, with an overview of the concordance and discordance relations used for building the outranking relations.

In section 3, the problem of evaluating a set of alternatives on a hierarchical criteria structure in ELECTRE III method is considered. Currently, proposed ELECTRE methods require that all criteria are defined on a common level and do not accept a hierarchy of criteria. This section

presents several possible adaptations of the ELECTRE III method to deal with hierarchical structure of criteria (ELECTRE III-H).

Section 4 covers the implementation of the proposed ELECTRE III-H methods and their application to the problem of managing the disposal of sewage sludge generated during the water cleaning process in wastewater treatment plants in the SOSTAQUA research project. This is a multiple criteria ranking problem with a hierarchical structure of criteria where results are expected from different levels by various teams of multi-disciplinary researchers involved in the project.

In section 5, the result obtained from applying the new method to the SOSTAQUA project is presented with an analysis of the sensibility of the methods with respect to the parameters (indifference, preference and veto thresholds).

Section 6 concludes by summarizing the theoretical evaluation of the appropriateness of the ELECTRE III-H method proposed in this thesis and presents ideas for further research work.

2. State of the art

Multiple criteria decision aiding (MCDA) is widely used in decision making to improve the quality of decisions, making the process more explicit, rational and efficient, by using theory and methodology to treat complex problems encountered in business, engineering and other areas of human activity. In a decision making problem with multiple criteria, a unique optimal decision for the problem does not exist but rather many or even infinitely many decisions may be suitable for a given problem [13]. A classical problem in the field of MCDM is to build a preference relation on a set of alternatives that are evaluated on multiple criteria. The evaluation is often based on preferences expressed on each criterion whereby, each criterion may be assigned weights according to the relative importance and in some cases accompanied with veto thresholds. A traditional way of evaluating an ordered pair of alternative is to allocate a score $v(a)$ to each alternative a and assert that a is preferred to b if and only if $v(a) > v(b)$ [14]. Usually, the values $v(a)$ and $v(b)$ depends on the evaluations based on the n -criteria ($g_1, g_2, \dots, g_n$) such that, $v(a) = V(g_1, g_2, \dots, g_n)$. Though this kind of method result in a preference relation built with the property of transitivity, the definition of the aggregation function V may not always be an easy task [15].

In general, the overall goal is to determine a preference ordering among a number of available alternatives ($a_1, a_2 \dots a_m$). The decision-maker's preferences over alternatives depend on how well they perform according to a number of criteria ($g_1, g_2 \dots g_n$) that have been identified by the decision maker as issues in which a decision between alternatives should be made. A complex problem is usually characterized by disproportionate and conflicting criteria or objectives such as cost, performance, reliability, safety, productivity, affordability, and other factors that can have effect on the choice of alternatives in a decision making process. Initially, the alternatives are assessed according to each criterion separately. In other words, for each criterion g_j the decision-maker must provide a "score" for each option ($a_1, a_2 \dots a_m$), whether in cardinal or ordinal terms. Multi-criteria methods are then employed to combine the criteria scores obtained for the alternatives into an overall preference ranking or choice of alternatives. Many methods for performing multiple criteria choice or ranking has been suggested, each with its own informational requirements and mathematical properties [1].

MCDA methodologies can be classified into three main groups: multiple attribute utility theory (MAUT), outranking methods and interactive methods [16]. The outranking method is explored in this thesis with special attention to the well-known ELECTRE III MCDA method [17-19], with the aim of extending the ELECTRE III method to hierarchical multiple criteria

problem. In this section, the basic elements of multiple criteria decision aid and concepts are presented with a brief overview of outranking methods, ELECTRE methods and concordance and discordance aggregation preference principles.

2.1. Basic concepts of multiple criteria decision aid

Many technical terms have been used in works related to multi-criteria decision aid. The most common terms are explained below.

Alternative: An alternative denoted by “ a ” represent a choice of alternatives available to the decision maker, it can be more generally referred to as a potential alternative because it is deemed possible to be implemented and it deserves some interest in the decision making process [1]. Usually, the set of alternatives to be screened, prioritized and eventually ranked, is assumed to be finite, ranging from several alternatives to hundreds of alternatives that does not necessarily need to be stable but can evolve throughout the decision making process.

Criterion: A criterion is a tool constructed for evaluating and comparing potential alternatives according to a well-defined point of view [20]. Alternatives are evaluated according to a criterion, which results in performance levels that can be represented as an evaluation matrix for decision making analysis. In [21], a criterion is defined as a “real valued function on the set A of alternatives, such that it appears meaningful to compare two alternatives a and b according to a particular point of view on the basis of two values $g(a)$ and $g(b)$ known as the performance of alternatives a and b according to the criterion g ”. For each criterion, it is necessary to explicitly define the set of all the possible evaluations each criterion can lead to in order to allow for the comparisons of the alternatives. A complete order called the preference scale of the criterion “ g ” can then be defined from the result of the evaluations. This notion of preference scale is very important because the goal of the decision making process is to determine the preferred option among the alternatives. According to some point of view, different forms of criterion can be defined. If for all alternatives $a, b \in A$:

$a P_g b \Leftrightarrow g(a) > g(b)$, i.e. a is strictly preferred to b if and only if $g(a)$ is greater than $g(b)$ and

$a I_g b \Leftrightarrow g(a) = g(b)$, i.e. a is indifferent to b if and only if $g(a)$ is equal to $g(b)$.

The underlying preference structure is a complete pre-order structure, meaning that all states are comparable, and any difference leads to strict preference. Cases where small differences are considered as indifference, lead to a quasi-criterion defined with a threshold “ q ” by:

$a P_g b \Leftrightarrow g(a) - g(b) > q$, i.e. a is strictly preferred to b if and only if the largest difference between $g(a)$ and $g(b)$ is greater than q and it is compatible with an indifference situation,

$a I_g b \Leftrightarrow |g(a) - g(b)| \leq q$, i.e. a is indifferent to b if the absolute difference between $g(a)$ and $g(b)$ is less than or equal to q .

The quasi-criterion is characterized by an underlying preference structure in the form of a semi-ordered structure. To allow for a smooth transition from indifference to strict preference, it is possible to introduce a weak preference Q_g which represents hesitations between indifference and preference [22], and functions as a buffer zone. The introduction of weak preference leads to a new form of criterion known as pseudo-criterion [23] with two thresholds p and q defined by:

$a P_g b \Leftrightarrow g(a) - g(b) > p$, where p is the preference threshold used to obtain strict preference and

$a I_g b \Leftrightarrow |g(a) - g(b)| \leq q$ represent the indifference relation, while the weak preference is defined by,

$a Q_g b \Leftrightarrow q < g(a) - g(b) \leq p$, is obtained when the absolute difference between $g(a)$ and $g(b)$ falls within the range of p and q .

The underlying preference structure is a pseudo-order structure [16]. Within the indifference threshold a decision maker does not perceive any difference between two alternatives. Giving a value to the thresholds p (preference threshold) and q (indifference threshold) is not an easy task. These thresholds can be fixed numbers or variables depending on the criteria, $q(g(a))$ or $p(g(b))$. Simple method such as weighted sum use true criteria, whereas outranking methods, such as ELECTRE III and PROMETHEE use pseudo-criteria. The representation of different points of view (aspects, factors, characteristics) with a family of criteria is a delicate part of the decision problem formulation [23]. If possible, it is advisable to represent all the different aspects of the problem at hand with a consistent family of criteria while avoiding redundancy. A family of criteria is coherent or consistent [23], if it is exhaustive, monotonic (if a is seen as better overall than b , it is also true for c whose performance is at least as good as that of a for all

criteria), non-redundant (no superfluous criteria) and understood and agreed upon by each stakeholder.

Decision Maker (DM): A decision maker is the person or group with the responsibility and authority to make decisions based on knowledge of the consequences of choosing from the available alternatives. Decision theory such as those proposed in MCDA methods is used to assist decision makers in the process of decision making. In group decision making, problems cover several fields which require a number of interacting decision makers with different areas of specialization, to work together with different data in order to make compatible decisions [24].

2.2. Preference Modelling

Preferences are an essential and inevitable element in the lives of an individual or within an organization. The modeling of these preferences is a vital step in building models for a better understanding and representation of a given situation. It is used extensively in areas such as decision making, operations research, economics, psychology, sociology, computer science and many other fields. Models in which comparison of objects is often necessary in order to either establish if there is an order between the objects can be better handled by building a preference structure over the family of criteria. The usual convention for building a preference model in MCDA assumes a numerical scale for the criteria where a criterion can be regarded as better than another if it is more than the other on the numerical scale.

Given a set of ordered pairs of alternatives (a,b) , the following three relations are defined for modeling preference between the given pair of alternatives:

Preference (P): $aPb \Rightarrow a$ is **preferred** to b ,

Indifference (I): $aIb \Rightarrow a$ is **indifferent** to b and,

Incomparability (J): $aJb \Rightarrow a$ is **incomparable** to b .

The preference model is said to be accomplished if the three relations: preference, indifference and incomparability fulfill the following requirements [16]:

$\forall a, b \in A$:

$$\left\{ \begin{array}{ll} aPb \Rightarrow b \narrow Pa: b \text{ is not preferred to } a & : P \text{ is asymmetric} \\ aIa \Rightarrow a \text{ is indifferent to itself} & : I \text{ is reflexive} \\ aIb \Rightarrow bIa \text{ (} a \text{ and } b \text{ are indifferent)} & : I \text{ is symmetric} \\ a \narrow Ja \Rightarrow a \text{ is incomparable to itself} & : J \text{ is irreflexive} \\ aJb \Rightarrow bJa \text{ (} a \text{ and } b \text{ are incomparable)} & : J \text{ is symmetric} \end{array} \right.$$

The three relations $\{P, I, J\}$ form a *preference structure* on the set of alternatives A , if the conditions above are satisfied and, given any two elements a, b in A , one and only one of the following properties is true: aPb, bPa, aIa, aJb . Different preference structures, some of which will be employed in later chapters, can be used with the above definition as a base for building the preference models.

2.3. Outranking methods

Outranking methods were developed in the sixties through the influence of French MCDA theoreticians as an alternative to methods using the value functions approach, they have been proposed for aggregating preference information on several criteria or performance measures into an overall preference structure in cases where multi-attribute utility approach is not appropriate or feasible. In [16] Vincke states that “*the underlying idea of introducing the outranking methods is that it is better to accept a result less richer than that yielded by MAUT, if one can avoid mathematical hypotheses which are too strong and requiring complex information from the DM*”.

The main aim of these methods is to build binary outranking relations S on the set A of alternatives, obtained by a pairwise comparison of the alternatives based on each criterion in the family of criteria, to produce a result richer than the dominance relations, and less rich but more realistic than multiple attribute utility or value functions [25].

Outranking methods have developed rapidly during the last few decades because of their adaptability to the poor structure of most real decision problems. They have been widely applied to major projects in many important fields including engineering, political science, economics and other related projects [26-28]. Outranking methods have been very successful because they are easy to understand and quite adaptable to real world problems. They require fewer assumptions about DM's preferences which made them more realistic are interactive than single criterion approaches. They also possess the advantage of being able to combine different criteria

scales (e.g., numerical, ordinal, linguistic), without the need to transform the data to a common domain. This characteristic is very important in certain real applications.

Their drawbacks, which can be checked by sensitivity analysis, are the arbitrary definitions of outranking and the setting of the threshold parameters [29].

One of the first outranking methods to appear in literature, in the late 1960s [16, 30] explored ways to provide formal foundations to known outranking methods, by suggesting a common framework to describe them. However, the theoretical framework of outranking methods changed in the nineties [31], demonstrating its diversity through a vast range of applications with stronger foundation [32]. On theoretical grounds, outranking method attempts to establish the theory of non-compensatory preference structures which offer more freedom to the decision maker to express preferences on performance measures in a structured manner, based on a pairwise comparison of alternatives [33].

Decision problems where one or few satisfactory alternatives are required by a DM is best handled by outranking methods because of their ability to adjust the DM's preferences to precision with very little information required from the DM [32].

In [30] Pirlot states that *“outranking methods are uniquely characterized by the limited degree to which a disadvantage on a particular criterion may be compensated for by advantages on another criterion (i.e. non-compensatory nature)”*. This means that, when two alternatives are compared, ‘small’ differences in favor of one of them may be compensated for by ‘small’ differences in favor of the other one, but ‘large’ differences may not be compensated for. Such a feature leads to overall preferences in which some pairs are termed incomparable.

In general, given two alternatives a and b in the set A , outranking relation between a and b can be defined as “alternative a outranks alternative b if, given the decision maker's preferences, the quality of the evaluation of the alternatives and the context of the problem, there are enough arguments to decide that alternative a is at least as good as alternative b , while there is no overwhelming reason to refute that statement” [29]. One alternative outranks another if it outperforms the other on enough criteria of sufficient importance and it is not outperformed by the other option by having a significantly inferior performance on any one single criterion. Each alternative in the set of alternatives is compared to all the other members of the set in a pairwise manner to determine their degree of outranking or outranked.

2.3.1. Outranking Concept

In [34] Roy defines an outranking relation of two alternatives a and b , as a binary relation S defined on a set of alternatives A , such that aSb (a outranks b) if, given what is known about the DM's preferences, and given the evaluations on alternatives and the nature of the problem, there exist enough arguments to decide that a is at least as good as b , while there is no essential reason to disapprove that statement.

While the above outranking concept is a general idea but not a precise mathematical definition for building outranking relation, Vincke [16] argues that it is not necessary for an outranking relation to be complete or transitive, but it may as well define a partial pre-order or ranking.

The concept of pseudo-criterion (criterion with preference thresholds) is used in some outranking methods to express the DMs' preferences on performance measures in such a way that, for each criterion, an indifference threshold, q , and a preference threshold, p , is defined ($p > q$). These thresholds are used to compare the evaluations of the alternatives a and b , relative to that criterion. Most of these methods also involve a notion of 'weights' for representing the relative importance of the criteria.

Outranking methods comprise two steps:

Step 1: Building the outranking relation, and

Step 2: Exploiting the relationship with special focus on the chosen statement of the problem

Outranking methods differ, among other aspects; by the way the above concepts are formalized for each method. Some of the most popular outranking methods are listed below with their references.

- a. Elimination Et Choix Traduisant la REalite' (ELECTRE) [34].
- b. Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) [35].

- c. Organization, Rangement Et Synthese de dones relaTionnelles (ORESTE) [36].
- d. Traitement des Alternatives Compte Tenu de l'Importance des Critères(TACTIC) [37].
- e. Expected value method EVAMIX [38]
- f. REGIME [39].
- g. Multicriterion Analysis of Preferences by means of Pairwise Alternatives and Criterion comparisons (MAPPAC) [40]

The outranking concept as used in ELECTRE methods is presented in the following section since ELECTRE III is the main focus in this thesis.

2.4. ELECTRE methods

In 1966, the ELECTRE (Elimination and Choice Translating REality) method was initiated by three French scholars (Benayoun, Roy and Sussmann) as an outranking method for evaluating a multiple criteria decision aid problem. Since then, important efforts have been made by scholars and students working to improve this method and publishing their different proposals involving theories and applications in worldwide articles [41]. Due to the concerted efforts of many theoreticians in this field, different versions of the ELECTRE method proposed include ELECTRE I, ELECTRE IS, ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE TRI. Though the ELECTRE evaluation method is widely considered as an effective and efficient decision aid with a broad range of applications, the selection of the most relevant version for a given decision problem is a key factor that can influence a decision making process. Thus, the standard for choosing the ELECTRE method that provides the most precise response to different issues is crucial. Otherwise, the resulting recommendations may define an objective that falls below the DM's expectations.

Since ELECTRE methods are based on the principle of outranking, the steps for constructing the outranking relation follow the basic step for building an outranking relation. Firstly, the evaluation method is required to establish preference or outranking relation, followed by a consistent exploration and analysis of the relation to support the decision making process. The implementation of the ELECTRE methods differ from one version to another according to the degree of complexity, quality of information (i.e. how rich the information is) or according to the nature of the decision problem. In the following sections, a brief description of ELECTRE

I, the earliest and simplest of the ELECTRE method is presented to provide a basis for understanding the underlying concepts, followed by an overview of the extensions, ELECTRE I, II, III, IV and TRI, for the purpose of introducing the concepts of veto thresholds and pseudo-criteria that are fundamental in the application of ELECTRE methods for multiple criteria decision aid.

2.4.1. ELECTRE I

ELECTRE I is one of the earliest multi-criteria evaluation method developed among outranking methods. The major purpose of this evaluation method is to select a desirable alternative from a subset F of alternatives based on two indices, the concordance index and the discordance index defined for each pairs of alternatives a and b such that, any alternatives which is not in F is outranked by at least one alternatives in F . The concordance index $c(a,b)$ sometimes referred to as the respect of the majority, measures the strength of the information that support the hypothesis that a is at least as good as b and the discordance index $d(a,b)$ sometimes referred to as the respect of minorities, measures the strength of evidence against the hypothesis.

Following the basic construction step for outranking methods, the first step in *ELECTRE I* involve the building of the outranking relation followed by the exploitation of the outranking relation. This method can be easily applied to choice problem whose criteria scores are given as ordinal and nominal scales.

2.4.1.1. Building of the Outranking Relation

Given a set A of alternatives and an ordered pair (a,b) in A , evaluated by a set of m criteria $(g_1, g_2, \dots, g_n)$. For each criterion with the following attributes:

a “weight” w_j increasing with the relative importance of the criterion g_j .

a veto threshold $v_j(g_j) > 0$.

The concordance index $c(a,b)$ is calculated for each ordered pair $(a,b) \in A$ by

$$c(a,b) = \frac{1}{W} \sum_{j: g_j(a) \geq g_j(b)} w_j, \quad (2.1)$$

Where

$$W = \sum_{j=1}^n w_j \quad (2.2)$$

$c(a,b)$ takes values between 0 and 1 as a measure of the arguments in favor of the assertion “ a outranks b ” such that, higher values indicate stronger evidence in support of the claim.

The discordance index is obtained by the use of the veto threshold defined for each criterion. Among the criteria in favor of option b , if the score for b on any one of these criteria is greater than the score for option a on the same criterion, with a value greater than or equal to the veto threshold, it is assumed that there is evidence strong enough to refute the assertion that “ a outranks b ” [42]. That is:

$$d(a,b) = \begin{cases} 1 & \text{if } g_j(b) - g_j(a) > v_j(g_j) \text{ for any } j, \\ 0 & \text{otherwise} \end{cases} \quad (2.3)$$

Thus, with the combination of concordance and discordance indices for criterion g_j , an outranking relation aSb can then be defined by the relation:

$$aSb \text{ iff } \begin{cases} c(a,b) \geq \hat{c}, \\ d(a,b) \leq \hat{d}, \end{cases} \quad (2.4)$$

Where $\hat{c}$ can be defined as a relatively large concordance threshold and $\hat{d}$ (if necessary) as a relatively small discordance threshold.

2.4.1.2. Exploitation of the outranking relation

The outranking S leads to a relation that can be represented by a graph in which vertices represent alternatives and edges joining two vertices represent the outranking relation between two alternatives. A subset F of alternatives can then be obtained such that:

$$\begin{cases} \forall b \in A \setminus F, & \exists a \in F: aSb \\ \forall a, b \in F, & a \not S b. \end{cases} \quad (2.5)$$

The aim is to find a subset F of incomparable alternatives that outranks at least any one alternatives that is not in F . This type of set is referred to as a kernel and there exist procedures to determine it. If the graph obtained presents no circuit, the kernel exists and it is unique. However, if the graph contains circuits, the kernel is not necessarily unique and may not exist. A technique to reduce the circuits is to replace each circuit by a unique element. This technique may lead to loss of some information contained in the outranking relation. This led to the design

of ELECTRE I is to mitigate the inconvenience of considering all alternatives as indifferent when cycles are formed in the graph. Hansen et al proposed another technique of dealing with this situation by introducing the concept of minimum weakness quasi-kernel [43].

2.4.2. ELECTRE II [44]

ELECTRE II, developed by Roy and Bertier [44] shortly after ELECTRE I, differs from ELECTRE I in that it is designed to rank alternatives rather than indicate the most preferred alternative. The outranking relation is built the same way as in ELECTRE I but instead of one outranking relation, two outranking relations are built based on two concordance thresholds s^1 and s^2 where, $s^1 > s^2$ and $s^1, s^2 \in [0.5, 1 - \min_{j \in J} w_j]$. The two outranking relations result in what is referred to as the *strong outranking* (situation where outranking is not disputable) and *weak outranking* (situation where outranking generate doubt).

Given an ordered pair (a, b) in the set of alternatives A , evaluated on a set of criteria $(g_1, g_2, \dots, g_n)$ where each criteria is assigned a weight w_j , expressing the relative importance of criterion g_j . The concordance index $c(a, b)$ whose value vary between 1 and 0 is calculated as in ELECTRE I equation (2.1) and (2.2).

The two concordance levels s^1 and s^2 are then chosen to generate the outranking relations S^1 (*strong outranking*) and S^2 (*weak outranking*) as follows:

for $i = 1, 2$

$$c(aS^i b) \geq s^i \text{ and } c(aS^i b) \geq c(bS^i a). \quad (2.6)$$

The effect of introducing the condition $c(aS^i b) \geq c(bS^i a)$ is to reduce the possibility of obtaining two alternatives outranking each other.

The exploitation of the two outranking relations can be carried out either with the top down method, which starts with the “best” alternative and works downwards in descending order or the bottom up method, which starts with the “worst” alternative and works up in ascending order. In the top down method, like in ELECTRE I, the class of best alternatives say, H which

are not strongly outranked by any other alternative is obtained from the circuits of S^1 and within H , the circuit of S^2 are reduced and the set say K of alternatives which are not weakly outranked by any other member are determined. This defines the first class of the descending ranking. The alternatives in K are then deleted and the exploitation procedure is repeated until all alternatives have been classified to obtain the ranking in descending order. The bottom up procedure is built starting with the class of worst alternatives (alternatives which outranks no other alternatives). This is obtained by determining the set of alternatives, say W , that do not strongly outrank any other alternative and within W , the set of alternatives say Z , which do not weakly outrank any other member of W is obtained for the first class of the ranking in ascending order. The elements in Z are discarded and the procedure is repeated until all the alternatives are classified to produce the ascending order ranking. The two procedures *ascending* and *descending* order generate what is strictly referred to as *complete preorders* or *weak orders* because no preference ordering is implied or inferred between alternatives in the same class.

It is worth noting that the two preorders obtained do not produce the same result, for example, an alternative which is not outranked and does not outrank any other alternative will appear as first on the list of the descending ranking and it will appear as last in the list of the ascending ranking. To handle this problem, it is advisable to build the partial preorder with the combined result of the ascending and descending ordering procedures. Several ways have been suggested to combine the two orders to obtain a single order of incomparable set of alternatives [45].

2.4.3. ELECTRE III [46]

ELECTRE I and II described above assume that all criteria are “*true*” criteria in the sense that any difference in performance correspond to a difference in preference. Assumptions are made such that indifference occurs only when two alternatives perform identically on a given performance. ELECTRE III is a ranking method designed to introduce the notion of *pseudo criteria* instead of *true-criteria* as in ELECTRE II method. It is less sensitive to inaccuracy, imprecision, wrong determination or uncertainty of data.

2.4.3.1.

Building of the Outranking Relation

Given an ordered pair (a, b) in the set A of alternatives, evaluated on a set of n pseudo-criteria $\{(g_j, q_j, p_j), j = 1, \dots, n\}$ where, each criterion g_j is assigned a weight w_j (expressing the importance of the criterion) and a veto threshold $v_j(g_j) > 0$. The construction of the outranking relation requires the definition of a credibility index, which characterizes the credibility of the assertion “ a outranks b ”, (aSb) defined by the concordance index $c(aSb)$, and a discordance index $d(aSb)$ for each criterion g_j . The concordance index is computed for each ordered pair (a, b) of alternatives as follows.

$$c(a, b) = \frac{1}{W} \sum_{j=1}^n w_j c_j(a, b) \quad , \quad (2.7)$$

Where

$$W = \sum_{j=1}^n w_j .$$

And

$$c_j(a, b) = \begin{cases} 1 & \text{if } g_j(a) + q_j(g_j(a)) \geq g_j(b) , \\ 0 & \text{if } g_j(a) + p_j(g_j(a)) \leq g_j(b) , \\ \frac{p_j(g_j(a)) - (g_j(a) - g_j(b))}{p_j - q_j} & \text{if } g_j(b) - p_j \leq g_j(a) \leq g_j(b) - q_j \end{cases} \quad (2.8)$$

where p_j and q_j denote the indifference and the preference thresholds respectively, used for the construction of the concordance index for each criterion. The definition of the concordance for the ordered pair (a, b) evaluated on criterion g_j is illustrated by the figure below.

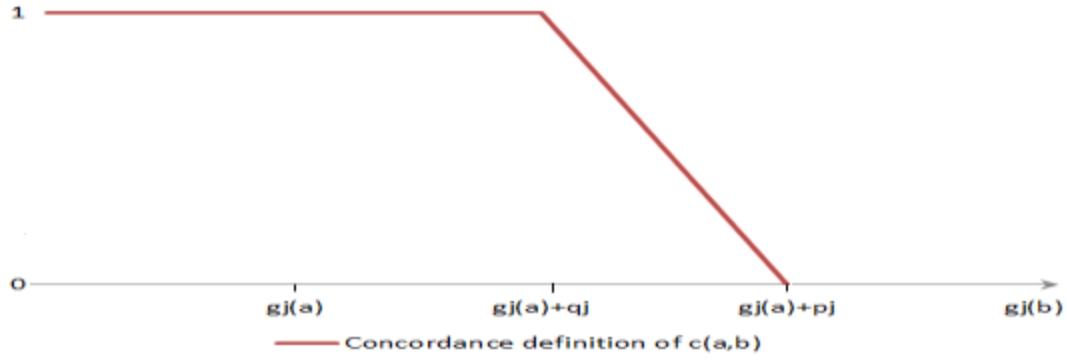


Figure 1 Concordance evaluated on criterion g_j for ordered pair (a,b)

If the criteria g_j are *quasi-criteria* (i.e. $q_j(g_j(a)) = p_j(g_j(a))$, $\forall a$ and $\forall j$), then, the concordance index becomes

$$c(a,b) = \frac{1}{W} \sum_{j: g_j(a) + q_j(g_j(a)) \geq g_j(b)} w_j = \frac{1}{W} \sum_{\substack{j: a P_j b \\ \text{or } a I_j b}} w_j, \quad (2.9)$$

Where (P_j, I_j) is the semi-order structure associated with criterion g_j .

The computation of the discordance takes criteria that disagree with the assertion aSb into account. It is defined by the introduction of a *veto threshold* $v_j(g_j)$ for each criterion, such that, the outranking of b by a is vetoed if the performance of b exceeds that of a by an amount greater than the veto threshold. The discordance index is obtained mathematically as follows:

$$d_j(a,b) = \begin{cases} 1 & \text{if } g_j(b) > g_j(a) + v_j(g_j(a)) \\ 0 & \text{if } g_j(b) \leq g_j(a) + p_j(g_j(a)) \\ \frac{g_j(b) - g_j(a) - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}, & \text{otherwise} \end{cases} \quad (2.10)$$

The overall concordance and discordance indices are then combined to obtain a valued outranking relation with credibility $\rho(aSb)$ defined by:

$$\rho(aSb) = \begin{cases} c(a, b) & \text{if } d_j(a, b) \leq c(a, b), \forall j \\ c(a, b) \cdot \prod_{j \in J(a, b)} \frac{1 - d_j(a, b)}{1 - c(a, b)} & \end{cases} \quad (2.11)$$

Where $J(a, b)$ is the set of criteria j for which $d_j(a, b) > c(a, b)$ and the degree of outranking is equal to the concordance index when there exist no discordant criterion. Valued outranking relation possesses the advantage that it is less sensitive to variation of the arbitrary values of the parameters.

2.4.3.2. *Exploitation of the outranking relation*

The exploitation procedure proposed for ELECTRE-III [ref] is an iterative distillation algorithm that selects at each step a subset of alternatives, taking into account the previously established outranking relations. It starts with a value $\lambda = \max_{a, b \in A} \rho(a, b)$ to compute such a binary relation in A that it is true for a credibility of outranking greater than λ , and false for a credibility of outranking smaller or equal to λ . This yields a crisp outranking relation for which the qualification $Q(a)$ of each alternative is computed (i.e. the number of alternatives which are outranked by a minus the number of alternatives which outrank a). This leads to the generation of a set of alternatives with the greatest qualification called the *first distillate* and denoted by D_1 .

If D_1 contains only one alternative, the procedure is repeated in $A \setminus D_1$. Otherwise, the same procedure is applied inside D_1 to obtain the *second distillate* D_2 ; if D_2 result in a singleton, the procedure is repeated again in $D_1 \setminus D_2$ (except if the latter is empty); otherwise, it is applied inside D_2 repeatedly until D_1 is completely used up, before starting with $A \setminus D_1$. Notice that it may happen that two or more alternatives belong to one distillate because they have the same qualification and neither of them can be ranked better or worse than others. In this case, the alternatives are said to be *indifferent* and are assigned to the same ranking position. This procedure, which yields a first complete preorder Z_1 is called the descending distillation chain.

A second complete preorder, Z_2 , is obtained by an ascending distillation chain, in which the alternatives having the smallest qualification are first retained. A final partial pre-order Z is then built as the intersection of the two complete pre-orders, Z_1 and Z_2 , which are obtained according to two variants of the same principle, both acting in an antagonistic way on the floating alternatives. The resulting ranking is a partial preorder, i.e. for any two alternatives from set A , one may be preferred over the other, or they may be indifferent, or they may be

incomparable. The incomparability of two alternatives occurs when one of these alternatives, say a , is ranked better than the other alternative, say b , in Z_1 (or Z_2), and b is ranked higher than a in Z_2 (or Z_1).

2.4.4. ELECTRE IV [47]

The ELECTRE IV method is designed as an extension of the ELECTRE III method with the aim of ranking the alternatives using a family of pseudo-criteria but eliminating the concept of weight introduced in previous versions of the ELECTRE methods.

Given an ordered pair (a, b) in the set A of alternatives, evaluated on a set of m pseudo-criteria $\{(g_j, q_j, p_j), j = 1, \dots, m\}$ where, each criterion g_j is assigned a veto threshold $v_j(g_j) > 0$, a strong and a weak outranking relation S^1 and S^2 respectively are defined as follows:

$$aS^1b \text{ if } \left\{ \begin{array}{l} g_j(a) + p_j(g_j(a)) \geq g_j(b), \forall j \\ \|\{j: g_j(a) + q_j(g_j(a)) < g_j(b)\}\| \leq \|\{j: g_j(b) + q_j(g_j(b)) < g_j(a)\}\| \end{array} \right. \quad (2.12)$$

$$aS^2b \quad \text{if } \{ g_j(a) + p_j(g_j(a)) \geq g_j(b), \forall j, \quad (2.13)$$

or

$$\text{if } \left\{ \begin{array}{l} \text{there exists exactly one criterion } k \text{ such that} \\ g_k(a) + v_k(g_k(a)) \geq g_k(b) > g_k(a) + p_k g_k((a)), \\ \|\{j: g_j(b) + p_j(g_j(b)) < g_j(a)\}\| > \frac{n}{2}, \end{array} \right. \quad (2.14)$$

Where $\|A\|$ denotes the number of elements in the set A .

A more refined variant based on five embedded outranking relations instead of two can be found in [48]. In the case of ELECTRE IV, many variants can be used depending on the context

of the application. Despite the fact that the weights of the criteria are not taken into consideration, the method can also be applied to a decision problem where the criteria have different level of importance.

The outranking relation is exploited using the same technique as in ELECTRE III but, simplified to a consideration of only two outranking levels. One of the levels is used to determine the subset D^I of alternatives with the largest qualifications in A for S^I (qualification of “ a ” refer to the number of alternatives outranked by a , minus the number of alternatives which outrank a). The two preorders are built exactly as in ELECTRE III and the information drawn is analogous to that obtained in ELECTRE II or ELECTRE III.

2.4.5. ELECTRE TRI [48]

ELECTRE TRI was designed to solve classification problems. The name ELECTRE TRI originated from the procedure used in the original version, where, alternatives are classified into one of three categories namely, acceptable, unacceptable and indeterminate. Since then, extension of the method has been proposed for use in classification problems with more than three different categories [49]. The ELECTRE TRI approach outlined in this section is based on the method proposed by Roy and Bouyssou.

Given a set A of alternatives, evaluated on a set of n pseudo-criteria $\{(g_j, q_j, p_j), j = 1, \dots, n\}$ where, each criterion g_j is assigned a weight w_j (expressing the importance of the criterion) and a veto threshold $v_j > 0$. The reference alternatives b^h are used for indicating the limits between the categories, by defining the vectors of their values for functions g_j , denoted by $(g_1^h, \dots, g_n^h)$ such that, each alternative is assigned to k predefined ordered categories as shown below:

$$g_j^h > g_j^{h-1}, \quad \forall j; \quad \forall h \in \{1, \dots, k\}. \quad (2.15)$$

For an alternative $a \in A$, an outranking relation is built on the set

$$\{a\} \cup \{b^h, h = 0, \dots, k\}. \quad (2.16)$$

The computation of the outranking relation can be carried out in different ways, one of which is, to emulate the method used in ELECTRE III where, only values greater than or equal to a certain level are considered. Other procedures for computing the outranking relations are the pessimistic and the optimistic assignment procedures. The pessimistic procedure assigns an alternative a to the highest category c^h such that a outranks b^{h-1} . On the other hand, the optimistic procedure assigns a to the lowest category c^f such that b^f strictly outranks a . A comparison of the optimistic and the pessimistic procedures with a detailed discussion of the theoretical properties and the practical aspect of the ELECTRE TRI method can be found in [48].

2.4.6. Concordance and Discordance relations in MCDA

With the proposal of ELECTRE I [34, 50], several MCDM techniques, one of which is the outranking methods have been proposed. The outranking method use a procedure based on a concordance discordance principle to build a preference relation [37, 44, 51].

The concordance and discordance principles are Multi-criteria classifiers derived from the well-known concepts of *majority* and *right of veto*, respectively [52], the basic idea is that, if a criterion can be associated to a formalized hypothesis or reason (for example “*a is at least as good as b*”) whereby, alternatives can be compared in a pairwise manner, then the concordance $c(a, b)$ represents the evaluation of the existence of positive reasons in favor of the hypothesis that “*a is at least as good as b*” and the discordance $d(a, b)$ represents the existence of significant negative reasons against the same hypothesis. The two conditions of *concordance* and *non-discordance* must be fulfilled to validate the assertion that “*a is at least as good as b*” or *a outranks b* denoted by aSb [48]. A simple and basic example of concordance and discordance as described by Roy in [34] and applied in ELECTRE I is defined in equation (2.1), (2.2) and (2.3). These formulations results in any sufficiently strong positive coalition of subset of important criteria whose sum is at least λ . A sufficiently strong negative coalition will be any single criteria endowed with the veto power to negate the hypothesis [53, 54]. Outranking methods are largely based on the above formulation with a number of possible variations which aims at defining the concordance and discordance relation using a large variety of formulas. It can be observed that the binary relation defined above is of little help on its own because it can only guarantee reflexivity [55] to produce an outranking relation that will not guarantee an ordering relation since completeness and transitivity cannot be guaranteed. Though, once the relation has been established, a further analysis called the exploitation procedure is used to transform the relation into an ordering relation (at least a partial order).

One major setback in the formulation is the question of fixing a precise value for q_j (the indifference threshold) because the concordance test can be sensitive to modifications of the value of the criterion [56]. A solution to this problem was proposed by B. Roy [46] where, a *concordance index* $c_j(a, b)$ was defined from the quantities $g_j(a)$ and $g_j(b)$ in the unit interval, for each criterion g_j and each ordered pair (a, b) . An example of this solution is the definition of the concordance indices proposed in the ELECTRE III method which is defined as follows:

$$c_j(a, b) = \frac{p_j(g_j(a) - \min\{g_j(b) - g_j(a), p_j(g_j(a))\})}{p_j(g_j(a) - \min\{g_j(b) - g_j(a), q_j(g_j(a))\})} \quad (2.17)$$

where q_j is an indifference threshold and p_j is a preference threshold.

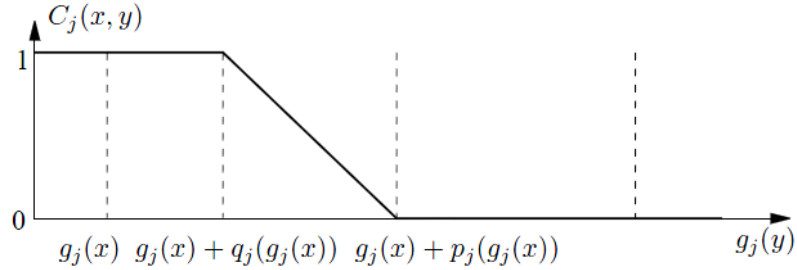


Figure 2 Valued outranking indices in ELECTRE III

Similar ideas have been proposed whereby, concordance indices are defined with respect to a strict preference $P(a, b)$ [57] and to indifference $I(a, b)$ [49]. In both cases, the concordance index can be considered as the membership degree of criterion g_j to the concordance coalition J_{ab}^S (or J_{ab}^P, J_{ab}^I).

Recent work showed that outranking relations can be defined to be an instance of non-transitive, non-additive conjoint measurement and a unifying frame with other approaches in MCDA called the “*Fuzzy Outranking Relation*” has been proposed. This new concept require the modification of the concordance test to incorporate changes whereby the concordant coalition with respect to $S(a, b)$ can be interpreted as a *fuzzy subset* with a membership function defined by $\mu_{J_{ab}^S}(j) = c_j(a, b)$. B. Roy suggested the following two ideas for the concordance test:

Adapt the ELECTRE I concordance tests to operate with concordance indices thereby, deriving the concordance test used in ELECTRE Is and defined by:

$$c(a, b) \Leftrightarrow \frac{\sum_{j \in J_{ab}^S} w_j c_j(a, b)}{\sum_j w_j} \geq \gamma \quad (2.18)$$

Interpret the concordance test in a multi valued logic as used in ELECTRE III [46]. In this case, the level of fulfillment of the concordance test can be defined as in equation (2.7).

The same idea is applied to the discordance test for criteria that strongly conflict with the hypothesis $S(a, b)$. The original idea of applying a veto threshold is not always convenient especially when the criterion scale is continuous; it is not always appropriate to assign a veto threshold to a given criterion over $S(a, b)$ when $g_j(b) - g_j(a) > v_j(g_j(a))$ should hold as soon as the condition is satisfied. Thus, a continuous transition is said to be preferable with discordance indices measuring the extent to which a criterion g_j strongly opposed the assertion $S(a, b)$.

$$d_j(a, b) = \min \left(1, \max \left(0, \frac{g_j(b) - g_j(a) - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))} \right) \right) \quad (j = 1, \dots, n) \quad (2.19)$$

Where $p_j(a) < v_j(a) \quad \forall a$.

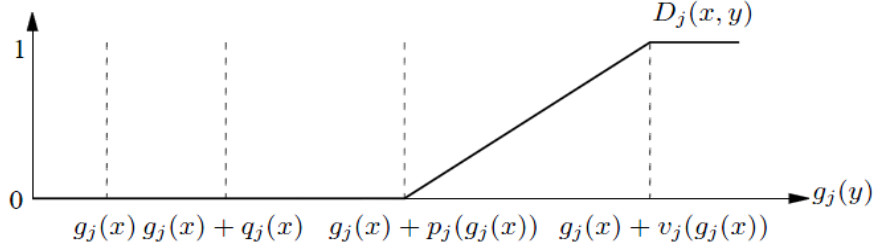


Figure 3 The discordance index in the ELECTRE III method

Based on the two ideas suggested by Roy, the discordance coalition can be modified with respect to $S(a, b)$ by incorporating a fuzzy subset membership function defined by

$$\mu_{J_{ab}}^D(j) = d_j(a, b) \text{ as follows:}$$

Adapt the ELECTRE I and ELECTRE III discordance test to operate with the discordance indices resulting in a simple solution inspired by the ELECTRE III method [46] as follows:

$$d(x, y) \Leftrightarrow 1 - \prod_{j \in J_{ab}^{D\gamma}} (1 - d_j(a, b)) > \delta \quad (2.20)$$

Where $J_{ab}^{D\gamma} = \{j \in \{1, \dots, n\}, d_j(a, b) > \gamma \text{ and } \gamma, \delta \in (0, 1) \text{ are the respective overall concordance and discordance thresholds. A single discordance criterion with } d_j(a, b) = 1 \text{ is sufficient to make the discordance test valid.}$

Interpret the discordance test in a multi valued logic as used implicitly in ELECTRE III [46]. In this case, positive discontinuities are avoided by using the threshold δ and the level of fulfillment of the discordance test can be defined as:

$$d(x, y) = 1 - \prod_{j \in J_{ab}^{D\gamma}} (1 - d_j(a, b)) \quad (2.21)$$

A more general and systematic construction of outranking relations using the fuzzy set theory can be found in [49, 58]. In general, when comparing two alternatives a and b using the concordance and discordance principle, any of the four different situations below can be produced:

- a) concordance and non-discordance
- b) concordance and discordance
- c) non-concordance and non-discordance
- d) non-concordance and discordance

The only valid situations among the four are that the outranking relation either holds or it does not hold.

Based on the concordance and discordance relation used by outranking methods, a generalized framework called the *NCD-based MCC methods (Nominal Concordance/Discordance based Multi-criteria classification)* has been proposed. The proposed NCD-based MCC methods are: *TRI-NOMFC classifier* [49], *PROAFTN classifier* [59], *PIP* and *K-PIP classifiers* [60] and *FBI classifier* [49]. The main idea of the NCD-based MCC is presented as follows:

Given the following notations:

A set of alternatives $A=\{a_i\}_{i=1,\dots,n}$, a set $G=\{g_j\}_{j=1,\dots,m}$ where each criterion g_j is assigned a weight w_j according to its relative degree of importance in the category C^h . a set $C=\{C^h\}$ of H nominal and predefined categories where each category C^h is characterized by a set of profiles or reference objects $B^h = \{b_k^h\}_{k=1\dots L_h}$ and the set of all profiles $B = \bigcup_{h=1}^H B^h$

The aim of NCD-based MCC is to compute a fuzzy number $\mu(a_i, C^h) \in [0,1]$ which measures the membership degree of each alternative a_i to a category C^h using the concordance and discordance concepts. For each criterion g_j , and for each profile b_k^h characterizing C^h , a local concordance $C_j(a_i, b_k^h)$ and discordance $D_j(a_i, b_k^h)$ indices are computed and combined such that, an alternative a_i is said to belong perfectly to a category C^h if $\mu(a_i, C^h) = 1$, and the alternative a_i does not belong to the category C^h if $\mu(a_i, C^h) = 0$. The formulas for the computation of the NCD-based MCC generalized framework are presented in the table below.

Method	Formula for Local and global concordance indices
PIP and K-PIP methods [60]	$C(a_i, b_k^h) = \sum_{j=1}^n w_j^h \times C_j(a_i, b_k^h) \text{ or } C(a_i, b_k^h) = \min_j C_j(a_i, b_k^h) \quad \text{where}$
	$C_j(a_i, b_k^h) = \begin{cases} 0 & \text{if } g_j(a_i) - g_j(b_k^h) \leq -p_j^h \\ \frac{1}{2} + \frac{1}{2} \sin(\pi/(p_j^h - q_j^h))(g_j(a_i) - g_j(b_k^h) + (p_j^h + q_j^h)/2) & \text{if } -p_j^h \leq g_j(a_i) - g_j(b_k^h) \leq -q_j^h \\ 1 & \text{if } g_j(a_i) - g_j(b_k^h) \leq q_j^h \\ \frac{1}{2} - \frac{1}{2} \sin(\pi/(p_j^h - q_j^h))(g_j(a_i) - g_j(b_k^h) + (p_j^h + q_j^h)/2) & \text{if } q_j^h \leq g_j(a_i) - g_j(b_k^h) \leq p_j^h \\ 0 & \text{otherwise} \end{cases}$
	$\text{and } \sum_{j=1}^n w_j^h$
	$= 1, \text{ with } q_j^h \text{ and } p_j^h \text{ representing indifference and preference thresholds respectively}$
	Formula for Local and global discordance indices
	$D(a_i, b_k^h) = 1 - \prod_j (1 - D_j(a_i, b_k^h)) \text{ or } D(a_i, b_k^h) = \max_j D_j(a_i, b_k^h) \quad \text{where}$
	$D_j(a_i, b_k^h) = \begin{cases} 1 & \text{if } g_j(a_i) - g_j(b_k^h) \leq -v_{jh}^- \\ \frac{1}{2} - \frac{1}{2} \sin(\pi/(v_{jh}^+ - v_{jh}^-))(g_j(a_i) - g_j(b_k^h) + (v_{jh}^+ + v_{jh}^-)/2) & \text{if } -v_{jh}^+ \leq g_j(a_i) - g_j(b_k^h) \leq -v_{jh}^- \\ 0 & \text{if } g_j(a_i) - g_j(b_k^h) \leq v_{jh}^- \\ \frac{1}{2} + \frac{1}{2} \sin(\pi/(v_{jh}^+ - v_{jh}^-))(g_j(a_i) - g_j(b_k^h) + (v_{jh}^+ + v_{jh}^-)/2) & \text{if } v_{jh}^- \leq g_j(a_i) - g_j(b_k^h) \leq v_{jh}^+ \\ 1 & \text{otherwise} \end{cases}$

	v_{jh}^- and v_{jh}^+ are the respective veto thresholds
PIP and K-PIP classifiers [60]	Formula for computing the similarity indices
	$SI(a_i, b_k^h) = C(a_i, b_k^h) \times (1 - D(a_i, b_k^h))$ $SI(a_i, b_k^h) = \text{Min}(C(a_i, b_k^h), 1 - D(a_i, b_k^h))$
Method	Formula for Local and global concordance indices
PROAFTN method [59]	$C(a_i, b_k^h) = \sum_{j=1}^n w_j^h \times C_j(a_i, b_k^h) \quad \text{where } C_j(a_i, b_k^h) = \text{Min}\{C_j^-(a_i, b_k^h), C_j^+(a_i, b_k^h)\},$ $C_j^-(a_i, b_k^h) = \frac{p_j^-(b_k^h) - \text{Min}\{S_j^1(b_k^h) - g_j(a_i), p_j^-(b_k^h)\}}{p_j^-(b_k^h) - \text{Min}\{S_j^1(b_k^h) - g_j(a_i), 0\}}, (a_i, b_k^h) = \frac{p_j^+(b_k^h) - \text{Min}\{g_j(a_i) - S_j^2(b_k^h), p_j^+(b_k^h)\}}{p_j^+(b_k^h) - \text{Min}\{g_j(a_i) - S_j^2(b_k^h), 0\}}$ $g_j(b_k^h) \text{ is defined by the interval } [S_j^1(b_k^h), S_j^2(b_k^h)], \sum_{j=1}^n w_j^h = 1 \text{ and } p_j^-(b_k^h) \text{ and } p_j^+(b_k^h) \text{ are two preference thresholds}$
	Formula for Local and global discordance indices
	$D(a_i, b_k^h) = 1 - \prod (1 - D_j(a_i, b_k^h))^{w_j^h} \quad \text{where } D_j(a_i, b_k^h) = \text{Max}\{D_j^-(a_i, b_k^h), D_j^+(a_i, b_k^h)\}$ $D_j^-(a_i, b_k^h) = \frac{g_j(a_i) - \text{Max}\{S_j^1(b_k^h) - p_j^-(b_k^h)\}}{p_j^-(b_k^h) - \text{Max}\{S_j^1(b_k^h) - g_j(a_i), v_j^-(b_k^h)\}},$ $D_j^+(a_i, b_k^h) = \frac{g_j(a_i) - \text{Min}\{g_j(a_i), S_j^2(b_k^h) + p_j^+(b_k^h)\}}{-p_j^+(b_k^h) - \text{Max}\{-S_j^2(b_k^h) + g_j(a_i), v_j^+(b_k^h)\}} \quad \text{where } \sum_{j=1}^n w_j^h = 1 \text{ and } p_j^-(b_k^h) \text{ and } p_j^+(b_k^h) \text{ are two preference thresholds and } v_j^-(b_k^h) \text{ and } v_j^+(b_k^h) \text{ are two veto thresholds}$
PROAFTN classifier [59]	Formula for computing the similarity index
	$SI(a_i, b_k^h) = C(a_i, b_k^h) \times (1 - D(a_i, b_k^h))$
FBI method [49]	Formula for Local and global concordance indices
	$C(a_i, b_k^h) = \sum_{j=1}^n w_j \times C_j(a_i, b_k^h) \quad \text{where } C_j(a_i, b_k^h) = \text{Min}\{S_j(a_i, b_k^h), S_j(b_k^h, a_i)\},$ $S_j = \frac{p_j - \text{Min}\{g_j(b_k^h) - g_j(a_i), q_j\}}{p_j - \text{Min}\{g_j(b_k^h) - g_j(a_i), p_j\}}, \text{ with } \sum_{j=1}^n w_j = 1, \text{ and } q_j$ $\text{and } p_j \text{ are indifference and preference thresholds respectively.}$
	Formula for Local and global discordance indices
	$D_I(a_i, b_k^h) = 1 - \prod_{j=1}^n (1 - D_j^I(a_i, b_k^h))^{a/n} \quad \text{where } D_j^I(a_i, b_k^h) = \text{Max}\{D_j^S(a_i, b_k^h), D_j^S(b_k^h, a_i)\},$

	$D_j^s(a_i, b_k^h) = \text{Min} \left\{ 1, \text{Max} \left\{ 0, \frac{g_j(b_k^h) - g_j(a_i) - p_j}{v_j - p_j} \right\} \right\}, \alpha \in [1, n]$ is a technical parameter introduced to modify the degree of synergy between criteria and v_j is the veto threshold.
FBI method [49]	Formula for computing the similarity index
	$SI(a_i, b_k^h) = \text{Min}(C(a_i, b_k^h), 1 - D(a_i, b_k^h))$
TRI-NOMFC method [13]	Formula for Local and global concordance indices
	$C(a_i, b_k^h) = \sum_{j=1}^n w_j^h \times C_j(a_i, b_k^h) \quad \text{where} \quad C_j(a_i, b_k^h) = F_j(g_j(a_i) - g_j(b_k^h)), F \text{ is an adaptation}$ of some PROMETHEE criteria function and $\sum_{j=1}^n w_j = 1$.
	Formula for Local and global discordance indices
	No discordance.

Table 1 Computation of the NCD-based MCC generalized framework

3. ELECTRE III-H proposals

The ELECTRE III multi-criteria outranking methodology is often used to assist a group of decision makers with different value systems to achieve a consensus on a set of possible alternatives. The normal ranking of ELECTRE III requires group decision problems to be structured in the form of a simple rectangular matrix (sometimes referred to as “flat structure”) of alternatives and criteria (Figure 3.1). However, for group decision problems involving decision makers at different levels, this method presents a weakness where evaluations on a subset of the criteria are often spread down into several levels of hierarchy. In order to adapt the ELECTRE III method to problems with hierarchical structure of criteria, a new methodology called ELECTRE III-H is proposed in this thesis, this method permits the decomposition of a multiple criteria ranking problem with hierarchical criteria into sub-problems corresponding to subsets of criteria with the same upper level root. ELECTRE III-H method adopts the concept of concordance and non-discordance tests used in ELECTRE III for evaluating and comparing the alternatives on each criterion to obtain a partial order at different sub-levels of the hierarchy; the partial orders resulting from each of these sub-levels are aggregated to obtain another partial ranking which is further propagated up the hierarchy until a final ranking of the alternatives is obtained.

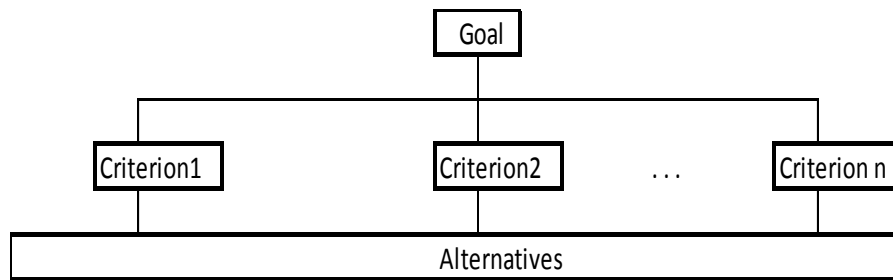


Figure 4 Example of a flat structure of criteria

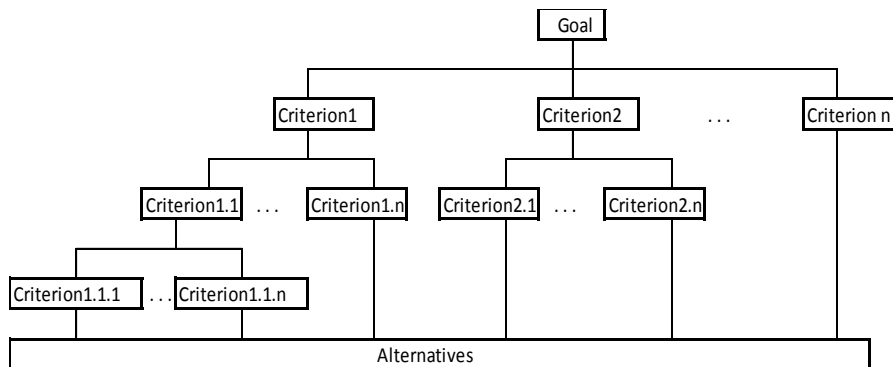


Figure 5 Example of a hierarchical structure of criteria

3.1. ELECTRE III-H methods

The ELECTRE III-H methods can be applied in some real applications, to solve decision making problems where the criteria is structured in a hierarchical way, distinguishing criteria at different levels of generality. In these cases, alternatives are compared in a pairwise manner based on their evaluation or score on each of the criteria at the lowest level, say $level_1$. The criteria at this level will correspond mainly to the most specific criteria used to evaluate the alternatives directly. Higher levels of the hierarchy, from $level_2$ up to the root level, which corresponds to the overall criteria, are consequently defined on the basis of the criteria at $level_1$. Based on the nature of the hierarchical structure of criteria, the ELECTRE III-H method comprises two main parts; the first part, referred to as the lowest level (*figure 3.2 criterion 1.1.1 to 1.1.n*) of the hierarchy, is computed using the outranking relation as applied in the original ELECTRE III method to evaluate the alternatives upon the sub-criteria at this level, followed by the exploitation of the outranking relation to obtain a partial order of alternatives. The results obtained at the lowest level are inherited by sub-criteria at upper levels in the form of ordinal scales. These scales are then used as inputs of the sub-criteria at the given upper level for computing the outranking relations.

This leads to the second part of the ELECTRE III-H method which serves as the main focus in the development of this thesis.

Figure 5 shows a graphical representation of a simple distribution of criteria in a hierarchical form. One major characteristic of this structure is that criteria from different nodes of the hierarchy may have different weights. If the ELECTRE-III method was applied in a node at $level_1$ to rank alternatives evaluated on criteria belonging to this node, the result would be a partial preorder Z . Consequently, to propagate the ELECTRE-III methodology to the next level $level_2$, we need to have a procedure to deal with partial preorders instead of total preorders.

In this section, we propose three extensions of the ELECTRE methods that permit the evaluation of partially ordered alternatives on upper-level criteria. This way, we will have the possibility of using the partial preorders inherited from lower levels of the hierarchy. It is assumed that the thresholds and the weights required for each terminal criterion of the hierarchy (i.e. leafs of the tree) are provided by the decision maker (DM) for the decision making process. With respect to the thresholds for intermediate level criteria, the user will not give any threshold because these are not final measurable criteria, but built in ones.

The basic information required in the application of the ELECTRE III-H methods is outlined below:

- a) a subset F of n hierarchically structured pseudo-criteria $\{(g_j, q_j, p_j), j = 1 \dots n\}$ with a common upper level root.
- b) a finite set of alternatives $\{a, b \dots\} \in A$ which is the same for all sub-problems resulting from the decomposition of the hierarchy.
- c) a weight w_j assigned to each criterion as a measure of the relative importance of the criterion

3.1.1. ELECTRE III-H with median preorder: Version1

In this section, we propose an extension of the definitions of concordance and discordance indices that permits the evaluation of partially ordered alternatives on upper-level criteria. We assume that the user has provided the indifference and preference thresholds for each terminal criterion of the hierarchical structure (i.e. leafs of the tree). The thresholds used for the intermediate level criteria are not final measurable values hence they will be built into the system and the user will not be required to provide further threshold values at this stage.

In general, each criterion in the hierarchy is considered as having a partially ordered scale. These scales of subsets of criteria F are generated at the lower levels and propagated to the criteria at upper levels. This way, we will have the possibility of using these results as performance of the alternatives for the corresponding intermediate levels criteria. The process is repeated progressively for each branch of the hierarchy until a final complete preorder of alternatives is obtained at the common root as explained in the sub sections below.

3.1.1.1. Building the Outranking relation at the lowest level

Given an ordered pair (a, b) in the set A of alternatives, evaluated on a set of n pseudo-criteria $\{(g_j, q_j, p_j), j = 1, \dots, n\}$ where, each criterion g_j is assigned a weight w_j (expressing the importance of the criterion) and a veto threshold $v_j > 0$. The first step involves the construction of the outranking relation which require the definition of a credibility index supporting the credibility of the assertion “ a outranks b ”, (aSb) defined by the concordance index $C(a, b)$, and a discordance index $d(aSb)$ for each criterion g_j .

The concordance index is computed for each ordered pair (a,b) of alternatives as follows:

$$c_j(a, b) = \begin{cases} 1 & \text{if } g_j(a) + q_j \geq g_j(b), \\ 0 & \text{if } g_j(a) + p_j \leq g_j(b), \\ \frac{p_j - (g_j(b) - g_j(a))}{p_j - q_j} & \text{if } g_j(a) + q_j \leq g_j(b) \leq g_j(a) + p_j \end{cases} \quad (3.1)$$

And

$$C(a, b) = \frac{\sum_{j=1}^n w_j c_j(a, b)}{\sum_{j=1}^n w_j}, \quad (3.2)$$

where $C(a, b)$ is the concordance index and p_j and q_j are the respective indifference and the preference thresholds, used in the construction of the concordance index for each criterion.

The computation of the *discordance index* requires the introduction of a *veto threshold* $v_j(g_j)$, for each criterion g_j , such that any credibility for the outranking of b by a is refused if at least one criteria disagree with the assertion aSb . The discordance index for each criterion is defined as follows:

$$d_j(a, b) = \begin{cases} 1 & \text{if } g_j(b) > g_j(a) + v_j \\ 0 & \text{if } g_j(b) \leq g_j(a) + p_j \\ \frac{g_j(b) - g_j(a) - p_j}{v_j - p_j} & \text{otherwise} \end{cases} \quad (3.3)$$

The degree of outranking is finally defined by:

$$S(a, b) = \begin{cases} c(a, b) & \text{if } d_j(a, b) \leq c(a, b), \forall j \\ c(a, b) \cdot \prod_{j \in J(a, b)} \frac{1 - d_j(a, b)}{1 - c(a, b)} & \end{cases} \quad (3.4)$$

Where $J(a, b)$ is the set of all criteria g_j for which the discordance indices are greater than the concordance indices (i.e. $d_j(a, b) > c(a, b)$).

This formula assumes that the concordance value would not be modified when the concordance exceeds that of the discordance. Otherwise, the assertion aSb is forced to be questioned and the value of $c(a, b)$ would have to be modified according to equation (3.4). This process leads to the generation of a credibility matrix that concludes the construction of the outranking model.

Notice that these definitions only need to make comparisons of values on a single criterion. This enables a smooth application of the method in cases involving heterogeneous data (e.g., numerical, ordinal, linguistic), which is a crucial issue in many applications.

3.1.1.2. Exploitation of the outranking relation at the lowest level

The next step is the exploitation of the model to produce a ranking of the alternatives from the credibility matrix following the general ELECTRE III procedure explained in section 2.4.3.2. The general approach for the exploitation is to construct two preorders Z_1 and Z_2 using the process of ascending and descending distillation to produce a partial preorder from the intersection of the two preorders ($Z_1 \cap Z_2$). The procedure involves the determination of a credibility value $\lambda = \max_{a, b \in A} S(a, b)$ from the credibility matrix such that only values of $S(a, b)$ that are sufficiently close to λ are considered, that is, $\lambda - s(\lambda)$, where $s(\lambda)$ is a threshold to

be determined. The credibility value is used to construct a matrix of the distillation processes with the definition of a new matrix T as follows:

$$T(a, b) = \begin{cases} 1, & \text{if } S(a, b) > \lambda - s(\lambda) \\ 0, & \text{otherwise} \end{cases} \quad (3.5)$$

For each of the alternative a qualification Q is defined such that, for a given alternative $a \in A$. $Q(a)$ refers to the number of alternatives that are outranked by project a minus the number of projects which outrank project a and it can be obtained as the row sum minus the column sum of the matrix T . The set of alternatives with the largest qualification is the *first distillate* of D_1 . If D_1 contains only one alternatives, the procedure is repeated in $A \setminus D_1$. Otherwise, the same procedure is applied inside D_1 to obtain the *second distillate* D_2 ; if D_2 result in a singleton, the procedure is repeated again in $D_1 \setminus D_2$ (except if the latter is empty); otherwise, it is applied inside D_2 repeatedly until D_1 is completely used up, before starting with $A \setminus D_1$. This procedure, which yields a first partial preorder Z_1 is called the descending distillation chain.

A second complete preorder, Z_2 , is obtained by an ascending distillation chain, in which the alternatives having the smallest qualification are first retained. A final partial pre-order Z is then built as the intersection of the two complete pre-orders, Z_1 and Z_2 , which are obtained according to two variants of the same principle, both acting in an antagonistic way on the floating alternatives. The resulting ranking is a partial preorder, i.e. for any two alternatives from set A , one may be preferred over the other, or they may be indifferent, or they may be incomparable. The incomparability of two alternatives occurs when one of these alternatives, say a , is ranked better than the other alternative, say b , in Z_1 (or Z_2), and b is ranked higher than a in Z_2 (or Z_1).

It is worth noting that the ranking at the lowest level of each branch of the hierarchy is computed according to the formulation given in this section, these preorders produced at the lowest level of the hierarchy are then propagated to upper levels of the hierarchy as explained In the following sections.

3.1.1.3. Building and Exploiting the Outranking relation at upper levels

The criteria at the upper level that have inherited results from the lower levels as explained in the previous section would have the scores associated to the ranking of each of the alternatives as performance values. Given a pair of alternatives (a, b) evaluated on an intermediate level criteria, the first step is to develop a measure of concordance (*concordance index* $C(a, b)$) for every pair of alternatives (a, b) with respect to the hypothesis that aSb . The concordance index aggregates the partial concordance indices $C_j(a, b)$ of criteria belonging to the same node of the tree.

The function $rank_j: A \rightarrow \mathbb{N}$ is defined to assign a score to an alternative according to its rank in the partial preorder inherited by any given node j , after applying ELECTRE-III on the criteria from this node. This score corresponds to the *rank* or *position* of an alternative, resulting from the application of the median order procedure to the downward and upward distillation, as applied in ELECTRE-III. The integer value scores are assigned such that, if the set of alternatives A contains m elements, the highest ranked (the best) alternative will have a score of m and the worst ranked of the alternatives will get a score of 1.

Being w_j the weight associated to criterion g_j , $rank_j(a)$ is alternative a 's score indicating the performance of a with respect to criterion g_j , q_j the indifference threshold and p_j the preference threshold. As mentioned earlier, the user is only expected to provide the indifference and preference thresholds for each terminal criterion (level₁) of the hierarchical structure. For the upper levels (level₂ upwards), we define the thresholds on the ranking positions as percentages of the total number of alternatives in A , such that: q_j marks the regions of indifference in the neighbourhood of each $rank_j$ score, while $p_j > q_j$ is the preference threshold on the $rank_j$ score. In this way, the thresholds are adapted to the maximum number of ranking positions. This definition of the percentages for the thresholds may not require the users input as they are designed to function internally within the hierarchy. Care should be taken in defining the percentages for the thresholds because a high percentage for a small number of criteria can greatly influence the final output of the decision process. For example, in case of 20 alternatives in A , $q_j=10\%$ and $p_j=20\%$ will set the values of q_j to 2 and p_j to 4. The values of q_j and p_j are rounded-off to integers.

The following *concordance index* $c(a, b)$ is defined for each pair of alternatives (a, b) :

$$c_j(a, b) = \begin{cases} 1 & \text{if } \text{rank}_j(a) \geq \text{rank}_j(b) - q_j \\ \frac{p_j - (\text{rank}_j(b) - \text{rank}_j(a))}{(p_j - q_j)} & \text{if } \text{rank}_j(a) + q_j \leq \text{rank}_j(b) \leq \text{rank}_j(a) + p_j \\ 0 & \text{if } \text{rank}_j(a) \leq \text{rank}_j(b) - p_j \end{cases} \quad (3.6)$$

The weighted sum of criteria at a given node is used to obtain the overall *concordance index* at the nodes as follows:

$$c(a, b) = \frac{\sum_{j=1}^n w_j c_j(a, b)}{\sum_{j=1}^n w_j}, \text{ where } w_j \text{ is the weight corresponding to criterion } j \quad (3.7)$$

The next step is to develop the measure of discordance with the assertion that a is at least as good as b . This should give the possibility of totally refusing the hypothesis that aSb by any discordant criterion. The discordance index is defined in terms of the rank_j score as follows:

$$d_j(a, b) = \begin{cases} 1 & \text{if } \text{rank}_j(a) + v_j \leq \text{rank}_j(b) \\ 1 - \frac{v_j - (\text{rank}_j(b) - \text{rank}_j(a))}{(v_j - p_j)} & \text{if } p_j < |\text{rank}_j(b) - \text{rank}_j(a)| < v_j \\ 0 & \text{otherwise} \end{cases} \quad (3.8)$$

Where $v_j > p_j > q_j$ is the veto threshold established as a percentage of the total number of alternatives

Next the *credibility index* is calculated for every pair of alternatives (a, b) according to ELECTRE III method as in equation 3.4.

The ranking of the alternatives is performed according to the *distillation procedures* explained in section 3.1.1.2.

This process is repeated, propagating the scores obtained from rankings at lower levels to the the upper levels where a final complete preorder is produced at the root level.

In cases where the results propagated to an upper level node the hierarchy depends on a combination of terminal and intermediate level criteria. This method permits the direct aggregation of these criteria with their performance values without the need for further computation.

3.1.1.4. Algorithm for ELECTRE III-H with median preorder

```

Step 1: For all first upper level nodes do {
    Collate all corresponding terminal criteria;
    For all criteria do{
        Calculate partial concordance index  $c_i(a,b)$  for each pair of alternatives;
        Calculate discordance index  $d_i(a,b)$  for each pair of alternatives;
    }
    Calculate global concordance index  $C(a,b)$  for each pair of alternatives;
    Compute credibility index with the combination of global concordance and discordance indices;
    // Exploit Outranking relation = Credibility index:
    Perform downward distillation procedure on outranking relation;
    Perform upward distillation procedure on outranking relation;
    Rank alternatives by applying the median preorder to downward and upward distillations;
    Score alternatives according to their positions in the ranking, with highest score for best alternative and lowest score for the worst alternative;
    Propagate scores to corresponding first upper level criteria;
}

Step 2: For all intermediate level node up to the highest level do {
    If corresponding criteria is a combination of terminal and intermediate level criteria then
        Collate all corresponding terminal and intermediate lower level criteria;
    Else
        Collate all corresponding intermediate lower level criteria;
    End if
    For all criteria do {
        Calculate partial concordance index  $c_i(a,b)$  for each pair of alternatives;
        Calculate discordance index  $d_i(a,b)$  for each pair of alternatives;
    }
    Calculate global concordance index  $C(a,b)$  for each pair of alternatives;
    Compute credibility index with the combination of global concordance and discordance indices;
    // Exploit Outranking relation:
    Perform downward distillation procedure on outranking relation;
    Perform upward distillation procedure on outranking relation;
    Rank alternatives by applying the median preorder to downward and upward distillations;
    If next upper level node is not highest level then
        Score alternatives according to their positions in the ranking, with highest score for best alternative and lowest score for the worst alternative;
        Propagate scores to the next corresponding upper level criteria;
    Else
        Final ranking is obtained at root node;
    End if
}

```

3.1.2. ELECTRE III-H by Net Flow Score ranking method: Version2

This version follows the ELECTRE III procedure proposed in Version1 except for the exploitation of the outranking relation where the Net Flow Score (NFS), which will be explained later in the thesis, is used instead of the distillation process to compute the complete pre-order at each node of the hierarchy. The adaptation of this version to problems with hierarchical structure of criteria permits the recommendation of the most preferred alternatives in the form of a complete order of alternatives at different levels of the hierarchy. In this case, criteria in the intermediate levels of the hierarchy are considered as having an ordinal, ordered scale inherited from the lowest level of the hierarchy. The process is repeated progressively for each branch of the hierarchy until a unique complete preorder of alternatives is produced at the common root.

The outranking process is carried out as follows:

3.1.2.1. *Building the Outranking relation at the lowest level*

Given an ordered pair (a, b) in the set A of alternatives and a set of n pseudo-criteria $\{(g_j, q_j, p_j), j = 1, \dots, n\}$ where, each criterion g_j is assigned a weight w_j and a veto threshold $v_j > 0$. The first step involves the construction of the outranking relation. This is achieved by using the concordance index $c(aSb)$, and a discordance index $d(aSb)$ to define a credibility index which supports the assertion “a outranks b” (aSb) for each criterion g_j . The formulation below is the same as in the usual ELECTRE III method and its reproduction is only for the purpose of clarity.

The concordance index is computed for each ordered pair (a,b) of alternatives as follows.

$$c_j(a, b) = \begin{cases} 1 & \text{if } g_j(a) + q_j \geq g_j(b), \\ 0 & \text{if } g_j(a) + p_j \leq g_j(b), \\ \frac{p_j - (g_j(b) - g_j(a))}{p_j - q_j} & \text{if } g_j(a) + q_j \leq g_j(b) \leq g_j(a) + p_j \end{cases} \quad (3.15)$$

And

$$C(a, b) = \frac{\sum_{j=1}^n w_j c_j(a, b)}{\sum_{j=1}^n w_j} , \quad (3.16)$$

where $C(a, b)$ is the concordance index and p_j and q_j are the respective indifference and the preference thresholds, used in the construction of the concordance index for each criterion.

The *discordance index* is computed with the introduction of a *veto threshold* v_j , for each criterion g_j , such that the credibility of the outranking of b by a is refused if at least one criteria disagree with the assertion aSb . The discordance index for each criterion is defined as follows:

$$d_j(a, b) = \begin{cases} 1 & \text{if } g_j(b) > g_j(a) + v_j \\ 0 & \text{if } g_j(b) \leq g_j(a) + p_j \\ \frac{g_j(b) - g_j(a) - p_j}{v_j - p_j} & \text{otherwise} \end{cases} \quad (3.17)$$

The degree of outranking is finally defined by:

$$S(a, b) = \begin{cases} c(a, b) & \text{if } d_j(a, b) \leq c(a, b), \forall j \\ c(a, b) \cdot \prod_{j \in J(a, b)} \frac{1 - d_j(a, b)}{1 - c(a, b)} & \end{cases} \quad (3.18)$$

Where $J(a, b)$ is the set of all criteria g_j for which the discordance indices are greater than the concordance indices (i.e. $d_j(a, b) > c(a, b)$). In cases where the concordance value exceeds that of the discordance, the concordance value will not be modified. By so doing, the credibility matrix is generated for the outranking model. The next step leading to the exploitation of the outranking relationship with the NFS approach is explained in the following sub-section.

3.1.2.2. Exploiting lowest level outranking relation using the Net Flow Score procedure (NFS)

Having obtained the outranking relation as in section 3.1.3.1 above, the resulting matrices can be represented by directed graphs, where nodes correspond to alternatives and arcs to outranking relations such that if an arc goes from node a to node b , then we can say that a outranks b (aSb), and the flow on this arc is equal to $S(a, b)$. In this case, arcs going from node a_i to other nodes will be represented by the row represented by the outranking relation of a_{ij} with other alternatives in the credibility matrix while, arcs entering the node a_i from other nodes will

be represented by the column of the credibility matrix corresponding to a_{ji} . To strengthen the degree of credibility of the assertion in the outranking graph, we take into account only those outranking relations which enjoy relatively high credibility, that is to say, if we define a cutting level δ such that $\delta = 0.5$, all entries of the credibility matrix $S(a,b)$ greater than or equal to 0.5 will be automatically set to 1 while values less than 0.5 will become 0.

In order to construct a complete preorder (ranking) of alternatives from the outranking graph using the Net Flow Score procedure, we compute the position of each of the alternatives using the balance of flows of the arcs entering and leaving a node. These positions will then be used to rank the alternatives from best to worst according to their NFS. The calculated balance is represented by the net flow score $NSF(a_i)$ which can be computed as follows:

$$NSF(a_i) = \sum_{j=1}^m a_{ij} - \sum_{j=1}^m a_{ji} \text{ where } i = (1, \dots, m) \text{ and } a_i \in A \text{ is an alternative} \quad (3.19)$$

Where $\sum_{j=1}^m a_{ij}$ is the sum of the rows of the matrix corresponding to alternative a in the credibility matrix and $\sum_{j=1}^m a_{ji}$ is the sum of the columns of the matrix corresponding to alternative a .

The net flow obtained for each of the alternatives is used to sort the alternatives into order where the alternative with the highest NFS is regarded as the best alternative and it will be ranked higher than any other alternatives in the set of alternatives whereas, the alternative with the least NFS will be ranked at the bottom of the complete preorder produced. Notice that ties can occur if the numbers of entering and leaving arcs are the same for two different alternatives.

3.1.2.3. *Building the Outranking relation at upper levels*

The outranking relation at upper level is built starting with a function defined by $rank_j: A \rightarrow \mathbb{N}$ which assigns an integer value score to each alternative, at any given intermediate node, according to the position in the complete pre-order propagated from the lowest level. This score corresponds to the *rank* or *position* of an alternative, resulting from the exploitation of the outranking relation by the NFS procedure.

The combination of these scores and the weights of each criterion at an intermediate node would be used to compute the outranking relations whereby the thresholds p_j , q_j and v_j can be defined as percentages of the number of alternatives in the set A of alternatives.

In a similar way to version 1, the *concordance index* $c(a,b)$ for each pair of alternatives (a,b) is calculated using the positions of the alternatives in the preorder inherited from the lower level by

$$c_j(a,b) = \begin{cases} 1 & \text{if } \text{rank}_j(a) \geq \text{rank}_j(b) - q_j \\ \frac{p_j - (\text{rank}_j(b) - \text{rank}_j(a))}{(p_j - q_j)} & \text{if } \text{rank}_j(a) + q_j \leq \text{rank}_j(b) \leq \text{rank}_j(a) + p_j \\ 0 & \text{if } \text{rank}_j(a) \leq \text{rank}_j(b) - p_j \end{cases} \quad (3.20)$$

The weighted sum of criteria at a given node is used to obtain the overall *concordance index* at the nodes as follows:

$$c(a,b) = \frac{\sum_{j=1}^n w_j c_j(a,b)}{\sum_{j=1}^n w_j}, \text{ where } w_j \text{ is the weight corresponding to criterion } j \quad (3.21)$$

The discordance index is defined in terms of the rank_j score as follows:

$$d_j(a,b) = \begin{cases} 1 & \text{if } \text{rank}_j(a) + v_j \leq \text{rank}_j(b) \\ 1 - \frac{v_j - (\text{rank}_j(b) - \text{rank}_j(a))}{(v_j - p_j)} & \text{if } p_j < |\text{rank}_j(b) - \text{rank}_j(a)| < v_j \\ 0 & \text{otherwise} \end{cases} \quad (3.22)$$

Where $v_j > p_j$ is the veto threshold on the rank_j score.

The complete pre-order at any given intermediate node will then be computed using the *NFS* procedure presented in section 3.1.2.2. This process is repeated by propagating the pre-orders obtained at lower levels to the upper levels, until a final complete pre-order is obtained at the root level.

There exists the possibility of a situation in the hierarchy where the result propagated to an upper level node depends on a combination of terminal and intermediate level criteria. In this case, this method permits the direct aggregation of these criteria with their performance values without the need for further conversion or computation.

3.1.2.4. Algorithm for ELECTRE III-H with Net Flow Score method

```

Step 1: For all first upper level nodes do {
    Collate all corresponding terminal criteria;
    For all criteria do{
        Calculate partial concordance index  $c_i(a,b)$  for each pair of alternatives;
        Calculate discordance index  $d_i(a,b)$  for each pair of alternatives;
    }
    Calculate global concordance index  $C(a,b)$  for each pair of alternatives;
    Compute credibility index with the combination of global concordance and discordance indices;
    // Exploit Outranking relation = Credibility index:
    Compute outward flow using outranking relation;
    Compute inward flow using outranking relation;
    Compute Net Flow Score = outward flow – inward flow;

    Score alternatives according to their Net Flow Score, with highest score for best alternative and lowest score for the worst alternative;

    Propagate scores to corresponding first upper level criteria;
Step 2: For all intermediate level node up to the highest level do {
    If corresponding criteria is a combination of terminal and intermediate level criteria then
        Collate all corresponding terminal and intermediate lower level criteria;
    Else
        Collate all corresponding intermediate lower level criteria;
    End if
    For all criteria do{
        Calculate partial concordance index  $c_i(a,b)$  for each pair of alternatives;
        Calculate discordance index  $d_i(a,b)$  for each pair of alternatives;
    }
    Calculate global concordance index  $C(a,b)$  for each pair of alternatives;
    Compute credibility index with the combination of global concordance and discordance indices;
    // Exploit Outranking relation = Credibility index:
    Compute outward flow using outranking relation;
    Compute inward flow using outranking relation;
    Compute Net Flow Score = outward flow – inward flow;

    If next upper level node is not highest level then
        Score alternatives according to their Net Flow Score, with highest score for best alternative and lowest score for the worst alternative;
        Propagate scores to the next corresponding upper level criteria;
    Else
        Final ranking is obtained at root node;
    End if
}

```

3.1.3. ELECTRE III-H by Credibility Propagation (CRED) method: Version3

In this section, we propose a solution for extending ELECTRE III to families of criteria organized hierarchically. It is based on propagating the outranking relations obtained using the concordance and discordance indices at the lowest level of the hierarchy to criteria on the upper-levels. This way, the respective credibility matrices inherited from the criteria at lower-levels of the hierarchy are aggregated using the weights of the criteria at corresponding higher levels. This process is repeated up the hierarchy using the combination of the credibility matrices and the weights until a final partial preorder is obtained at the common root. This approach takes the name CRED as it is based on credibility propagation.

We assume that the indifference and preference thresholds for each terminal criterion of the hierarchical structure (i.e. leafs of the tree) will be provided by the user. With respect to the intermediate criteria, the user will only need to provide information concerning the weight of each criterion for the aggregation process in the upper levels.

3.1.3.1. *Building the Outranking relation at the lowest level of the hierarchy*

The computation of the degree of outranking (credibility matrix) at the lowest level of the hierarchy is the same as in version 1 and 2. This process completes the first phase of this version of the ELECTRE III-H method.

Assuming that A is the finite group of n possible alternative solutions and m the number of the evaluation criteria at a given level on the hierarchy ($j = 1, 2 \dots m$). For the lowest level, we define a valued outranking relation $S(a,b)$ as the outranking credibility of a over b taking its values between 0 and 1. The value of $S(a,b)$ is defined based on the so-called concordance and discordance indices. A concordance index $c(a,b)$ is computed for each pair of alternatives (a, b) by

$$c(a,b) = \frac{\sum_{j=1}^n w_j c_j(a,b)}{\sum_{j=1}^n w_j} , \quad (3.9)$$

Where w_j ($j=1 \dots m$) is the weight of each criterion and

$$c_j(a, b) = \begin{cases} 1 & \text{if } g_j(a) + q_j \geq g_j(b), \\ 0 & \text{if } g_j(a) + p_j \leq g_j(b), \\ \frac{p_j - (g_j(b) - g_j(a))}{p_j - q_j} & \text{if } g_j(a) + q_j \leq g_j(b) \leq g_j(a) + p_j \end{cases} \quad (3.10)$$

A discordance index $d_j(a, b)$ is defined for each criterion g_j by

$$d_j(a, b) = \begin{cases} 1 & \text{if } g_j(b) > g_j(a) + v_j \\ 0 & \text{if } g_j(b) \leq g_j(a) + p_j \\ \frac{g_j(b) - g_j(a) - p_j}{v_j - p_j}, & \text{otherwise} \end{cases} \quad (3.11)$$

Where p_j is the preference threshold and v_j is the veto threshold of criterion j .

The degree of outranking is finally defined by

$$S^L(a, b) = \begin{cases} c(a, b) & \text{if } d_j(a, b) \leq c(a, b), \forall j \\ c(a, b) \cdot \prod_{j \in J(a, b)} \frac{1 - d_j(a, b)}{1 - c(a, b)} & \end{cases} \quad (3.12)$$

Where $S^L(a, b)$ is the outranking relation at the lowest level.

3.1.3.2. Building the Degree of Credibility at upper levels of the hierarchy

Given the credibility matrices inherited from a lower or lowest level of the hierarchy, this phase involves the propagation of these matrices to the next upper-level and further up the hierarchy until a final credibility matrix is obtained at the root node.

Taking into consideration that the weight of a criterion in ELECTRE methods underlines the relative importance of this criterion in the coalition of criteria (to which this criterion belongs), which are in favor of the hypothesis that a outranks b (aSb). We defined another credibility function $S_j^U(a, b)$ and a threshold $\hat{S}$ such that any value in the inherited credibility matrix ($S^L(a, b)$) greater than or equal to $\hat{S}$ will get a score of 1 while values less than $\hat{S}$ will get a score of 0. Then, the weight is multiplied by the value of $S_j^U(a, b)$ on any criterion j and the output of this operation is combined for all the criteria at the corresponding nodes to obtain a global

credibility matrix for the node according to equations (3.13) and (3.14). The outcome of $S_j^U(a, b)$ refers to the credibility of outranking of a over b inherited from the lower level; this credibility is coded by 0-1, and thus it plays the role of a concordance index of criterion g_j^U , so the multiplication of this 0-1 concordance by the weight of criterion g_j^U and addition over all criteria from level U having the same parent criterion at level $U+1$ is like an aggregation in a voting procedure, where voters (criteria) have different strength. In this way, we avoid the use of the weights at level U as substitution rates, which is completely different is like an aggregation in a voting procedure, where voters (criteria) have different strength. This way, we avoid the use of the weights at level U as substitution rates, which is completely different from the interpretation of weights as strengths of voters. Substitution rates express precise rates of compensation between components and based on the formulation of ELECTRE methods, the use of direct substitution rates will lead to a combination of models with the lowest level being non-compensatory, and the upper level compensatory, leading to a loss of all the advantages of ELECTRE-type non-compensatory aggregation.

$$S_j^U(a, b) = \begin{cases} 1 & \text{if } S^L(a, b) \geq \hat{S} \\ 0 & \text{if } S^L(a, b) < \hat{S} \end{cases} \quad (3.13)$$

where $0 < \hat{S} < 1$ is a built in threshold

$$S^U(a, b) = \frac{\sum_{j=1}^n w_j S_j^U(a, b)}{\sum_{j=1}^n w_j} , \quad (3.14)$$

Where $SU(a, b)$ is the *credibility matrix* at an upper level, $S_j(a, b)$ is the credibility matrix inherited from a node at the lowest level S^L or an intermediate node at a lower level S^U ; w_j is the weight assigned to the corresponding criteria at the given sub-level. Notice that this procedure does not require further definition of thresholds at higher levels of the hierarchy besides that used at the lowest level.

In a situation where the performance values of the alternatives on upper level criteria depend on the combination of terminal and intermediate criteria, this method requires that the credibility

matrices of the terminal criteria be obtained, using the usual ELECTRE III procedure, before the aggregation process can be completed.

The exploitation of the final credibility matrix is carried out by performing the ELECTRE III distillation procedures at the root node. The alternatives are ranked based on their qualification from the best to the worst (descending distillation process) and from the worst to the best (ascending distillation process). The final ranking of the alternatives at higher levels is built based on the application of the median preorder on the downwards and upwards distillation.

In common with ELECTRE III method, the partial preorder for the descending distillation is obtained by determining the maximum value of the credibility index, $\lambda_{max} = \max S(a,b)$, where this maximum value is taken over the current set of alternatives under consideration. Another value λ^* is computed from this maximum value using the formula

$$\lambda^* = \lambda_{max} - (0.3 - 0.15\lambda_{max}) \quad (3.15)$$

For each alternative, the λ -strength is determined as values greater than λ^* which represents the number of alternatives in the current set to which the alternative under consideration is preferred to. These values are generated from the row of the credibility matrix with reference to each of the alternatives. Likewise, the λ -weakness is determined as the number of alternatives in the current set which are preferred to the currently evaluated alternative. In this case, the λ -weakness corresponds to the row of the given alternative. For each of the alternatives, the qualification is computed as the λ -strength minus the λ -weakness. This value obtained for the qualification is used to classify the alternatives into distillate such that the set of alternatives having the largest qualification is called the first distillate, D_1 . If D_1 has more than one member, the process is repeated on the set D_1 until all alternatives have been classified.

The ascending distillation is obtained in the same way as the descending distillation except that the qualifications forming the first distillate in the ascending distillation process are the set of alternatives having the lowest qualification.

The final phase of the exploitation of the credibility matrix is the application of the median order to the outcome of the two distillates to obtain a final preorder of the alternatives from the best alternative to the worst.

With the CRED approach, we perform a propagation of the credibility matrix rather than a partial order of alternatives to higher levels of the hierarchy. It has the advantage that it does not require the specification of the indifference, preference and veto thresholds at levels other than the lowest level of the hierarchy and the computation is less rigorous compared to that used in the first version.

On the other hand, the CRED method permits to have a ranking result only at the root level. So, the decision maker can only receive a recommendation of the most preferred alternatives at the root level rather than at different levels of the hierarchy as proposed in previous methods.

3.1.3.3 Algorithm for CRED method

```

Step 1: For all first upper level nodes do {
    Collate all corresponding terminal criteria;
    For all criteria do{
        Calculate partial concordance index  $c_i(a,b)$  for each pair of alternatives;
        Calculate discordance index  $d_i(a,b)$  for each pair of alternatives;
    }
    Calculate global concordance index  $C(a,b)$  for each pair of alternatives;
    Compute credibility index with the combination of global concordance and discordance indices;
    Propagate credibility matrix to corresponding first upper level criteria;

Step 2: For all intermediate level node up to the highest level do {
    If corresponding criteria is a combination of terminal and intermediate level criteria then {
        Collate all corresponding terminal and intermediate lower level criteria;
        For all terminal criteria do {
            Calculate partial concordance index  $c_i(a,b)$  for each pair of alternatives;
            Calculate discordance index  $d_i(a,b)$  for each pair of alternatives;
        }
        Calculate global concordance index  $C(a,b)$  for each pair of alternatives;
        Compute credibility index with the combination of global concordance and discordance indices;
        Collate credibility indices inherited from terminal and intermediate level criteria;
    }
    Else
        Collate all corresponding credibility indices inherited from intermediate lower level criteria;
    End if
    For each pair of alternatives do {
        Compute new credibility index with the combination of the weights of each criterion, the credibility indices and the threshold  $\hat{S}$ ;
    }
    If next upper level node is not highest level then
        Propagate credibility matrix to the next corresponding upper level criteria;
    Else {
        // Exploit Outranking relation:
        Perform downward distillation procedure on outranking relation;
        Perform upward distillation procedure on outranking relation;
        Rank alternatives by applying the median preorder to downward and upward distillations;
        Final ranking is obtained at root node;
    }
    End if
}

```


4. Application of ELECTRE III-H methods to Environment sewage sludge disposal

The residue of sewage sludge produced at wastewater treatment plants (WWTP) has been found very useful especially when applied on agricultural soils as fertilizer or combustible. As a consequence, governments encourage their citizens to reinforce the valorisation of these products by recycling their nutrients and organic matter.

In Spain the production of biosolids has increased by 39% from 1997 to 2005 [61]. The legislation requiring plants to be built in towns with a population of more than 2000 inhabitants has led to the construction of more than 1000 WWTP built in the whole of Spain.

Sewage sludge application on soil improves fertility and exempt fertilizers use. Furthermore the transfer of nutrients and organic matter to the soil increases crop production. For these reasons, the Spanish government calls for the application of at least 70% of the WWTP sludge to agricultural soils. In 2005, the recycling of sewage sludge to agriculture represented a 65% of the total disposal of biosolids [61]. However, how this percentage may be increased require some scientific investigation. Some studies have proved that sewage sludge application improves soil fertility along the years [62]. Even so, sewage sludge application on soils may lead to *groundwater* nitrification as a result of nitrogen movement through *lixivate*. The *nitrate*s content of sewage sludge is a variable related to application dose. Special care must be taken on areas that are vulnerable to nitrogen pollution, as recommended on EC Nitrate Directive 91/676.

In addition to these issues, the impacts to humans and ecosystems must be considered, such as the exposition to organic contaminants through different routes (inhalation, ingestion and dermal contact), the lifecycle, and the field properties.

4.1. Problem Specification

In the research work conducted at URV in the project SOSTAQUA (CDTI, Ingenio 2010 – CENIT) and the Spanish project (CTM2007-64490), we have studied the problem of obtaining a global suitability index that evaluates the impact produced by the application of a certain type of sludge.

The goal is to help the managers in evaluating the suitability of a particular type of sewage sludge in some particular soils. This requires the analysis of the physical and chemical properties of the sludge and the soils in order to calculate the degree of suitability of each case study, taking into account that some soil matrices are more suitable to receive some kind of sewage sludge than others. This way, a method to evaluate a set of different samples of sewage sludge coming from different WWTP is required alongside the availability of a set of soils for receiving sludge in a given area. The aim of the decision maker is to find the best possible distribution of the sewage sludge available in the soil samples of a given territory.

In this project a set of relevant criteria were defined, with the joint work of a multidisciplinary team [63]. The elicitation of the values on those criteria is not straightforward and different methodologies to obtain them are being proposed. Moreover, some of the information considered in the evaluation model is subjectively defined by a domain expert, so we can have different opinions from different people. The treatment of such uncertain information requires the use of methods that are able to deal with it.

The calculated indicators represent the suitability of each pair (sewage sludge, soil). A brief description of these indicators is given in the following table.

Criteria	Subcriteria	Description
Environmental	Biodiversity	The most suitable pairs are those who lead to a lower impact on soil biodiversity. To that, SS (heavy metals, POPs, treatment type) and soil (OM, texture, carbonates) parameters are considered in the evaluation.
	Nitrates	The most suitable pairs are those who lead to a lower soil contamination by nitrates. To that, SS (OM, treatment type, nitrates) and soil (texture, nitrates) parameters are considered in the evaluation.
	Organic matter	Soil organic matter regulates several processes in soil. The evaluated inputs are soil OM and SS OM and treatment type .
	pH	Metals availability on soils is related to its pH. In this regard, basic soils are preferred for SS amendment. Acid soils should receive SS with high pH. The evaluated inputs are soil pH and SS pH .
	Soil contamination	Evaluates soil contamination by heavy metals and POPs. The evaluated inputs are: heavy metals and POPs concentration is SS; treatment type, soil OM, texture, pH and carbonates .
	GWcontamination by metals (GW metals)	Evaluates the likelihood of GW contamination by heavy metals. The inputs are SS heavy metals and total N, treatment type and soil texture, pH, carbonates and organic matter .
	GWcontamination by nitrates (GW nitrates)	Expresses GW vulnerability to contamination by nitrates. The inputs are SS treatment type and total N , the GW vulnerability of the area.
Human exposure	Crop type	The exposure varies depending on the type of crop.
	Distance to urban areas	The exposure is lower at higher distances.
	Ingestion dose	The suitability is calculated considering sewage sludge (heavy metals, POPs, treatment type) and soil (OM) characteristics. Lower concentrations of contaminants and high OM lead to a higher suitability for this criterion. Also, higher values are attributed to more stable SS.
	Labour	Evaluates the exposure of the farmer according to SS treatment type and application type .
	Population density	The exposure is lower at lower population density.
	Population type	The human exposure is also related to population sensitivity. Three classes are considered: sensitive (elder people and children), mid-sensitive (general population), and non-sensitive (no human presence).
	Precipitation	Considers the mean annual precipitation. At higher precipitation levels, the concentration in soils and transference to plant is lower.
	Temperature	Considers the mean annual temperature. The higher is the temperature, the higher the degradation and lower the exposure.

Table 2 Definition of criteria

From the work done, a decision system was designed and implemented with the following three components [62]:

(1) The *filtering* of data to guarantee the fulfilment of the European legal regulations on chemical and organic components,

(2) A *fuzzy expert system* with a set of rules that indicates which soil characteristics are more appropriate according with each sewage sludge properties, and

(3) An *aggregation system* based on multi-attribute utility theory, which is used to calculate a global impact score for each alternative from about 20 criteria. The aggregation is done using the LSP methodology.

In this master thesis, we will substitute the last component, the LSP aggregation module, with different methods proposed in the thesis. Tests will be done with data obtained after the execution of the filtering and fuzzy expert system modules.

In particular a dataset with 4 different soils and 3 different sewage sludge samples will be analysed. The table below is a result extracted when the *fuzzy expert system* is used to combine the soil characteristics with the sewage sludge properties.

Soil_ID	Sludge_ID	Alternatives
L1	S1	A
L1	S2	B
L1	S3	C
L2	S1	D
L2	S2	E
L2	S3	F
L3	S1	G
L3	S2	H
L3	S3	I
L4	S1	J
L4	S2	K
L4	S3	L

Table 3 Table of Alternatives obtained with expert system

For simplicity, we will refer to the combinations of soil and sludge according to the letter of the corresponding alternatives (i.e. A, B... L).

4.2. Criteria Hierarchy

The criteria in the management of sewage sludge disposal problem presented above require a hierarchical structure where the family of criteria, with common root, are organized into different levels according to table 2. In the first level, two types of criteria are distinguished: *Environment* and *Human Health Risk*. Each of these two criteria constitutes different aspects which were decomposed into simpler, well-defined attribute measures. We refer to the criteria at the lowest level of the hierarchy as terminal criteria whose combination will result in the score for each alternative for each criterion in an intermediate level node. The architecture of the criteria hierarchy with weights of each criterion is as shown in figure 6. The weights are regarded as coefficients of importance and are like votes given to each of the criterion.

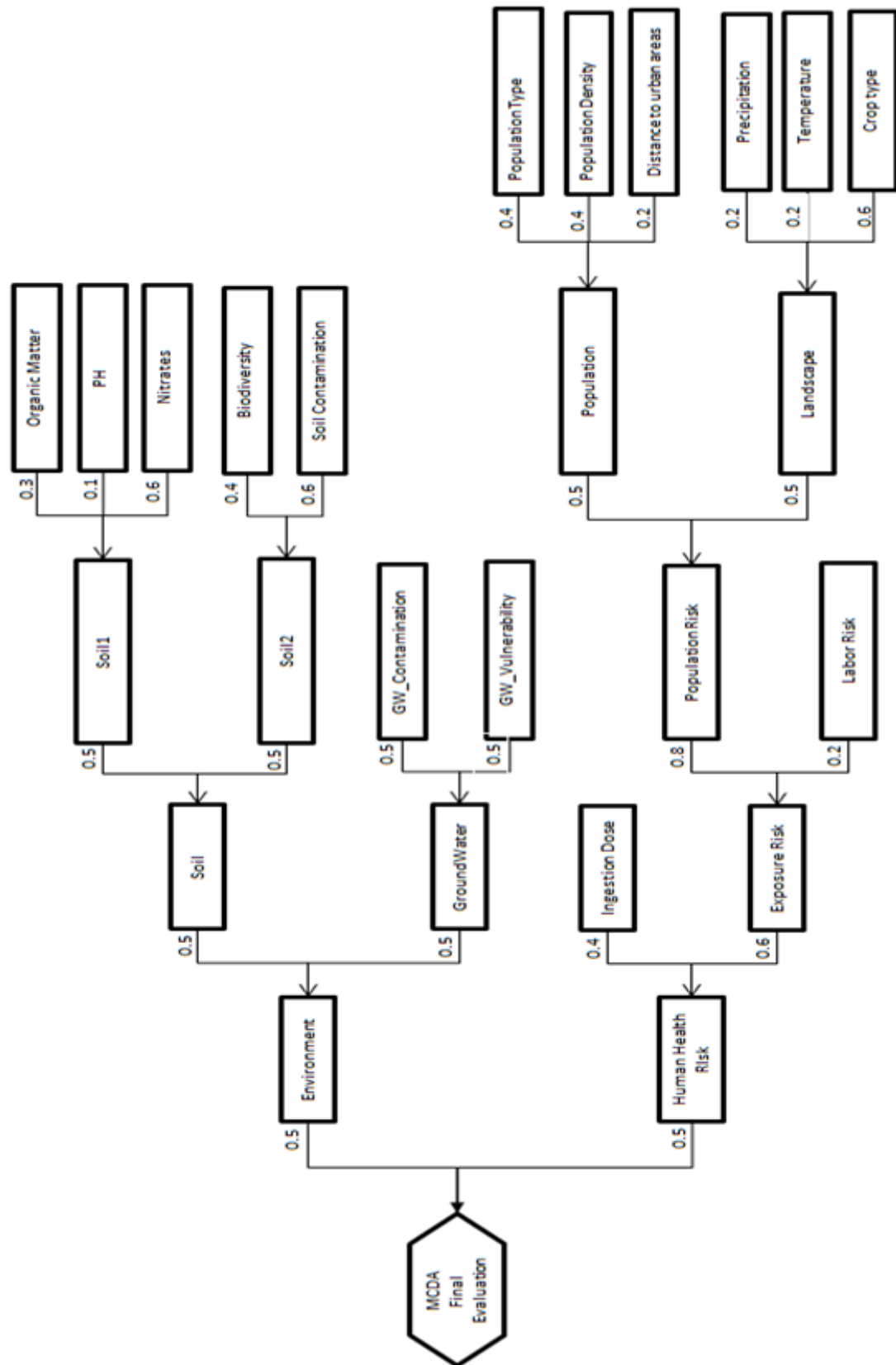


Figure 6 Hierarchical structure of criteria for sewage sludge disposals

4.3. Data

The data used in this test was created based on the studies made in the SOSTAQUA project where the alternatives are represented by different sludge and soil properties in Catalonia.

The scores for the criteria are arbitrarily scaled and defined by a number of attributes, not meaningful outside this application, but together describe the performance of the alternatives. These scores have different scales of measurement depending on their attributes. For instance, temperature is measured in degree Celsius, precipitation is in millimeters and distance is in kilometers. The values of the criteria, referred to as composite criteria, with scores in the range 0-1 were computed with the fuzzy expert system mentioned in step 2 of section 4.1. This system use rules to evaluate the characteristics of some attributes and their intersection to produce a final degree of suitability for these criteria on the 0-1 scale. This implies that the closer the degree of suitability of a criterion is, to 1, the better the criterion is.

The data used in the thesis are represented in the table below.

Alternatives	Temperature	Precipitation	Crop type	Population type	Population density	Distance to urban areas	GW contamination	GW vulnerability	Ingestion dose	Labor	Biodiversity	Nutrients	OM	pH	Soil contamination
A	10,00	125,00	1,00	1,00	0,50	7,00	0,64	0,40	0,63	0,40	0,50	0,68	0,75	0,97	0,37
B	10,00	125,00	1,00	1,00	0,50	7,00	0,60	0,50	0,70	0,60	0,53	0,68	0,80	0,80	0,60
C	10,00	125,00	1,00	1,00	0,50	7,00	0,55	0,50	0,52	0,20	0,33	0,70	0,65	0,97	0,41
D	7,50	100,00	0,90	2,00	0,19	4,00	0,71	0,40	0,70	0,80	0,51	0,65	0,75	0,97	0,43
E	7,50	100,00	0,90	2,00	0,19	4,00	0,37	0,50	0,83	0,97	0,52	0,64	0,65	0,90	0,62
F	7,50	100,00	0,90	2,00	0,19	4,00	0,54	0,50	0,50	0,60	0,41	0,70	0,70	0,97	0,44
G	8,00	180,00	0,60	2,00	1,00	4,00	0,64	0,40	0,63	0,40	0,61	0,75	0,75	0,97	0,48
H	8,00	180,00	0,60	2,00	1,00	4,00	0,60	0,50	0,70	0,60	0,55	0,76	0,80	0,80	0,63
I	8,00	180,00	0,60	2,00	1,00	4,00	0,55	0,50	0,52	0,20	0,53	0,83	0,65	0,97	0,50
J	6,00	200,00	0,30	3,00	0,13	15,00	0,62	0,40	0,52	0,80	0,56	0,74	0,75	0,97	0,40
K	6,00	200,00	0,30	3,00	0,13	15,00	0,47	0,50	0,60	0,97	0,52	0,72	0,90	0,80	0,46
L	6,00	200,00	0,30	3,00	0,13	15,00	0,38	0,50	0,50	0,60	0,43	0,63	0,65	0,97	0,44

Table 4 Performance table for sewage sludge disposals

Please note that criteria above have all been *maximised*, which means that the higher the score of a given criterion, the more preferable it is.

5. Tests and Results

For the purpose of applying the proposed methods to the data in section 4.3, the provision of the thresholds by the decision maker is required. In this case, the thresholds in table 5 represent subjective input provided for the purpose of the test. Care needs to be taken in determining threshold values, which must relate specifically to each criterion and reflect the preferences of a decision maker.

Thresholds	Temperature	Precipitation	Crop type	Population type	Population density	Distance to urban areas	GW contamination	GW vulnerability	Ingestion dose	Labor	Biodiversity	Nutrients	OM	pH	Soil contamination
q	1,00	20,00	0,15	0,00	0,10	1,00	0,08	0,00	0,05	0,20	0,05	0,02	0,01	0,00	0,08
p	1,50	30,00	0,35	1,00	0,30	4,00	0,15	0,09	0,10	0,30	0,10	0,05	0,12	0,12	0,10
v	5,00	500,00	0,50	2,00	0,60	10,00	0,30	0,30	0,20	0,50	0,20	0,15	0,20	0,15	0,25

Table 5 Thresholds for sewage sludge disposals

These thresholds are limits that are defined to establish cut-off points which aim at distinguishing regions of preference, indifference and refusal of an assertion. Within the family of ELECTRE methods, the three thresholds are defined below:

Indifference threshold (q): this is a value marking the point where an alternative is strictly preferred to another.

Preference threshold (p): this threshold defines an interval within which the difference of two alternatives is insignificant. This implies that a decision maker can freely make a choice between any two alternatives tagged as indifferent.

Veto threshold (v): this is a limit beyond which the credibility of the outranking relation of two alternatives is refused.

For the purpose of the thesis, more emphasis is placed on the computational process of aggregating and propagating the results obtained at lower levels of the hierarchy to higher levels of the hierarchy, up to the root level. As the objective of the thesis is to adapt the ELECTRE method to hierarchically structures criteria, there is no major variation in the computational process of the proposed methods. Thus, interested readers would find detailed information concerning the process of computing the original ELECTRE III method in [21, 22, 31]. To reduce the volume of data to the minimum, we included the results obtained at each level of the hierarchy and an explanation of the aggregation process at the nodes. Results of the

concordance and the credibility matrices can be found annexed to the document. To reduce the computational volume, we designed the computational process to combine the calculation of the discordance index with that of the credibility matrix in a single outranking matrix.

5.1. Application of version 1: ELECTRE III-H with median preorder

From the architecture of the hierarchy of criteria presented in figure 6, we will illustrate the application process of Version 1, to data provided for the management of sewage sludge disposals, with a breakdown of the hierarchy to several smaller parts. For this test, the *indifference*, *preference* and *veto* thresholds for terminal criteria corresponds to those provided for the test while all the intermediate level nodes are defined respectively as 15%, 25% and 50% of the total number of alternatives.

5.1.1. Ranking Human Health Risk criterion at the lowest level of the hierarchy

Ranking on the branch of criterion *Human Health Risk*, commenced with the application of the original ELECTRE III method formulation, described in sections 3.1.1 and 3.1.2 to the combined data of three terminal criteria, for each of the criteria, *population* and *Landscape*, at the lowest level. As a result, two complete preorder ranking of alternatives are produced with scores assigned to each of the alternatives based on their position in the ranking. These scores are translated into values measuring the performance of the alternatives with respect to each of the criteria *population* and *Landscape* at the next upper level of the hierarchy as shown in figure 7. For example, considering the criterion Population, alternative *G* has a score of 12 because it is recommended as better than all the other alternatives whereas alternative *E* has a score of 1 being the worse alternatives in the set of alternatives.

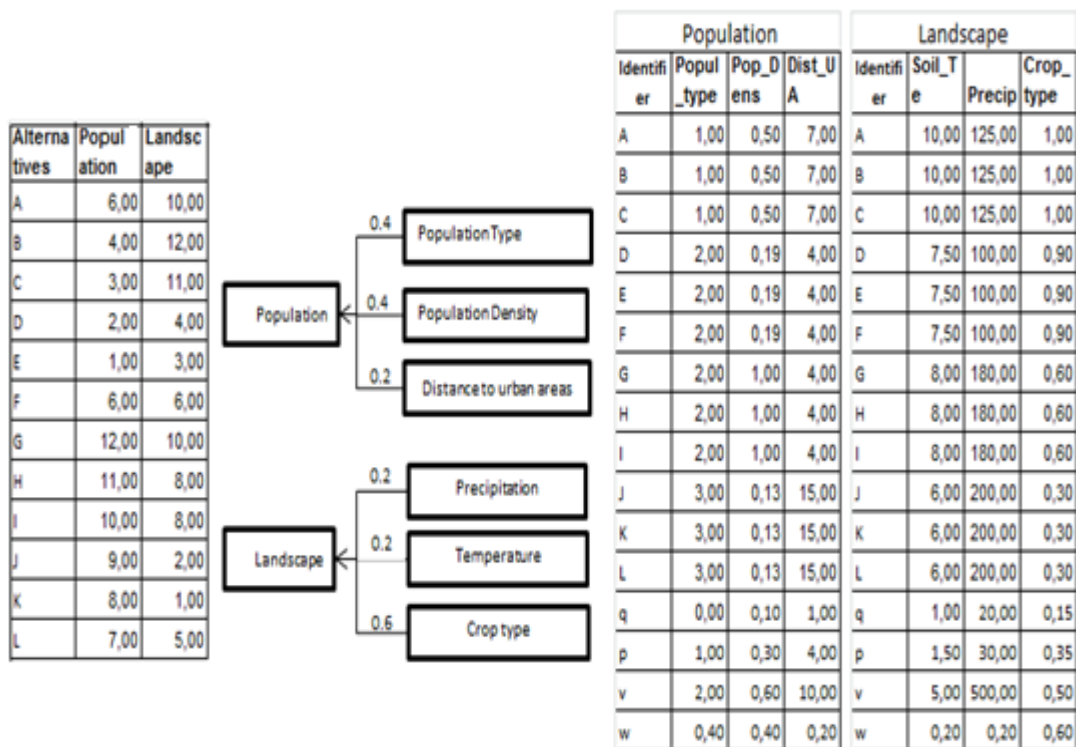


Figure 7 Ranking lowest level Branch of Human Health Risk criterion

5.1.2. Ranking Human Health Risk criterion at upper levels

The scores inherited from the lowest level for each of the alternatives translate into performance values of the criteria in the upper layer by which the alternatives are further evaluated with the adapted ELECTRE III-H method (section 3.1.3) at each of the upper level branches. Notice that the thresholds used at upper levels are designed to function internally and their definition does not necessarily require the input of the decision maker.

Figure 8 shows the ranking obtained after the application of the version to the combination of criteria; Population and Landscape at the first upper level. The result generates ranking which is transformed into scores corresponding to the position of the alternatives in the complete preorder.

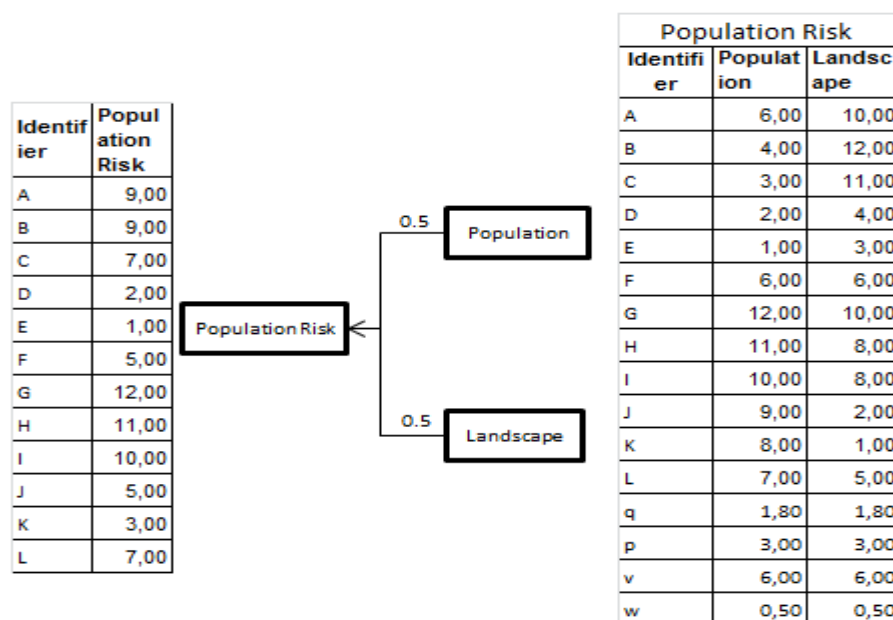


Figure 8 Human Health Risk criterion first upper level ranking

These scores are then used as performance values of the alternatives on the Population Risk criterion.

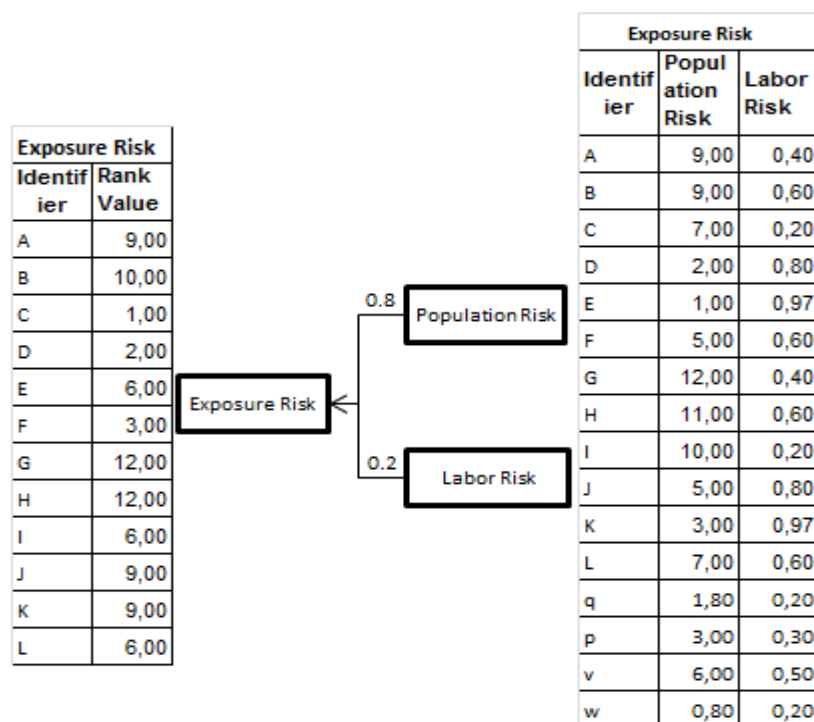


Figure 9 Human Health Risk criterion second upper level ranking

According to the architecture of the criteria hierarchy, the ranking inherited by *Population risk* is combined with the terminal criteria *Labor Risk* to produce a ranking which is propagated to the next upper level figure 9. Notice that the design permits the combination of terminal and intermediate level criteria. The same situation where a terminal criterion is combined with an upper level criterion is repeated at the second upper level. Here, the performance table inherited from the first upper level by the *Exposure Risk* criterion is combined with another terminal criterion, *Ingestion Dose*, to produce a ranking at the node corresponding to Human Health Risk criterion figure 10.

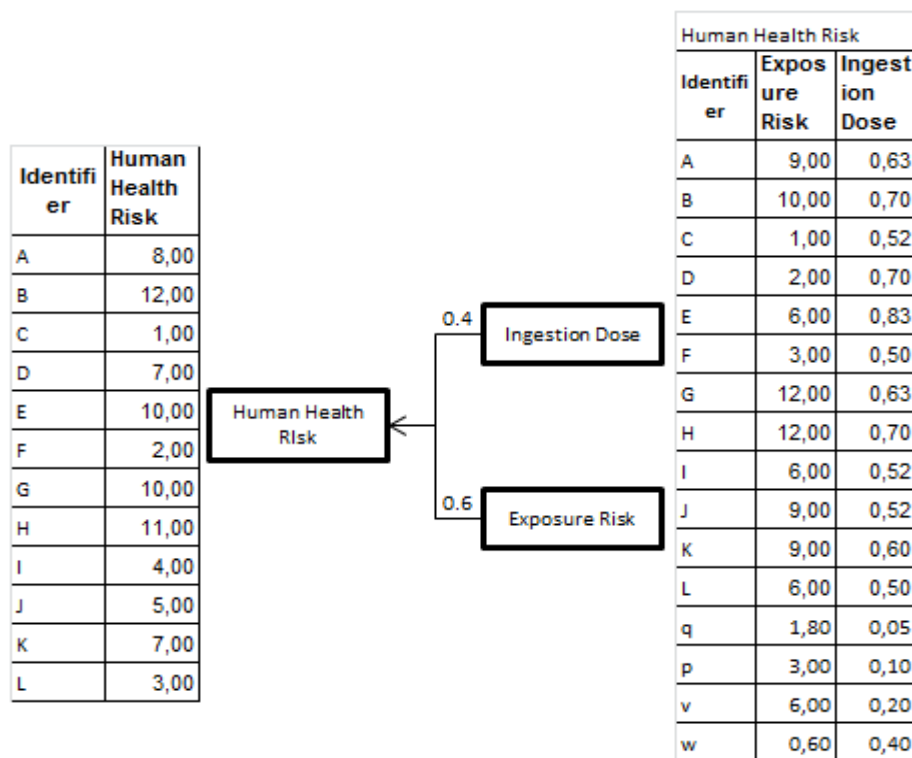


Figure 10 Human Health Risk criterion ranking

5.1.3. Ranking Environment criterion at the lowest level of the hierarchy

The ranking on the branch of the criterion *Environment* is carried out in two parts at the lowest level because it contains the channels leading to *Soil* and *Groundwater* whose results are later combined at upper levels. The procedure for ranking along the two branches is similar to that of *Human Health Risk*, starting with the application of the original ELECTRE III method

formulation, described in sections 3.1.1 and 3.1.2, to a combination of criteria at the lowest level as shown in figure 11 for Soil and figure 12 for Groundwater. The outcome of the two complete preorder on the branch of the *Soil* criterion facilitates the assignment of scores to the alternatives based on their position in the ranking. These scores are then translated into values and propagated to the next level as a measure of the performance of each of the alternatives with respect to the two criteria *Soil1* and *Soil2*.

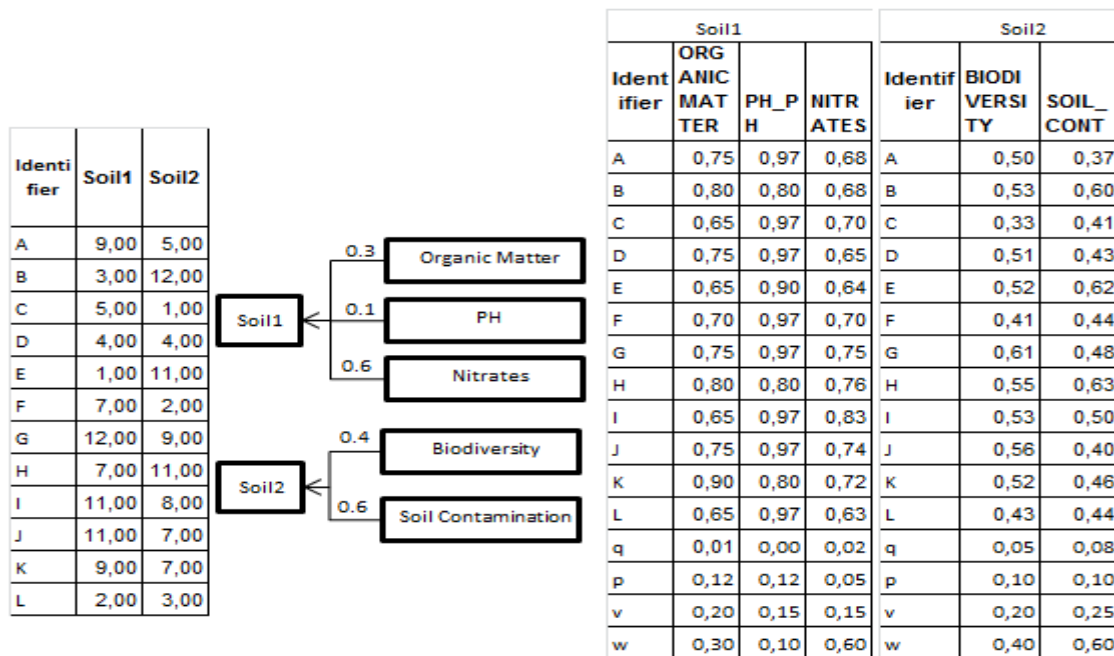


Figure 11 Ranking lowest level branch of Soil criterion in the Environment criterion branch

Likewise, the result of the complete preorder on the *Groundwater* branch generates the ranking for which scores are allocated to the alternatives for the criterion (Groundwater) as shown in figure 12.

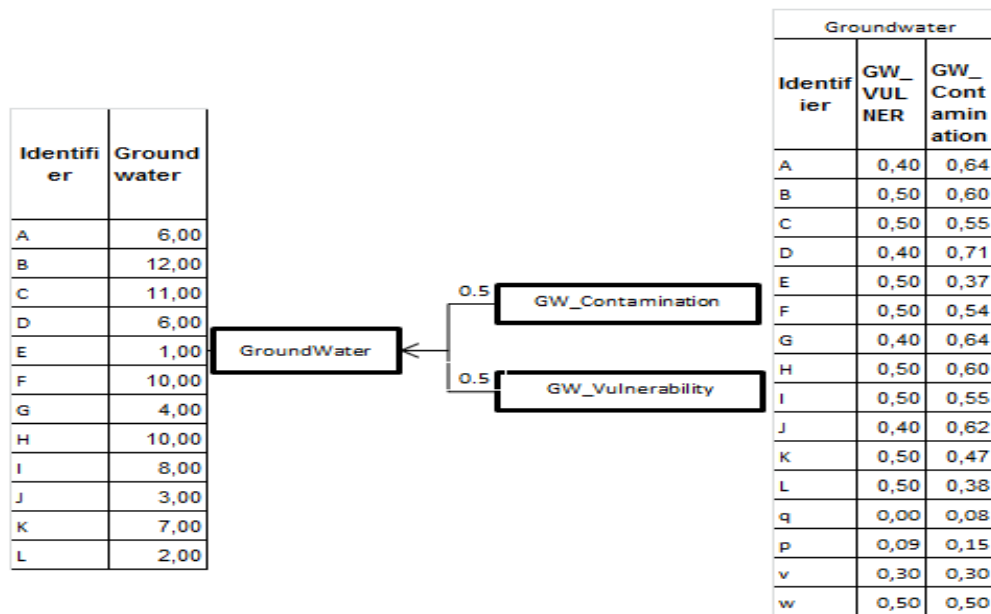


Figure 12 Ranking lowest level branch of Groundwater criterion in the Environment criterion branch

5.1.4. Ranking Environment criterion at upper levels of the hierarchy

The performance scores obtained for Soil1 and Soil 2 are combined at the first upper level using the adapted ELECTRE III-H method (section 3.1.3) to produce a ranking of complete preorder of alternatives which are subsequently taken as the performance values of the alternatives at the node corresponding to Soil, figure 13.

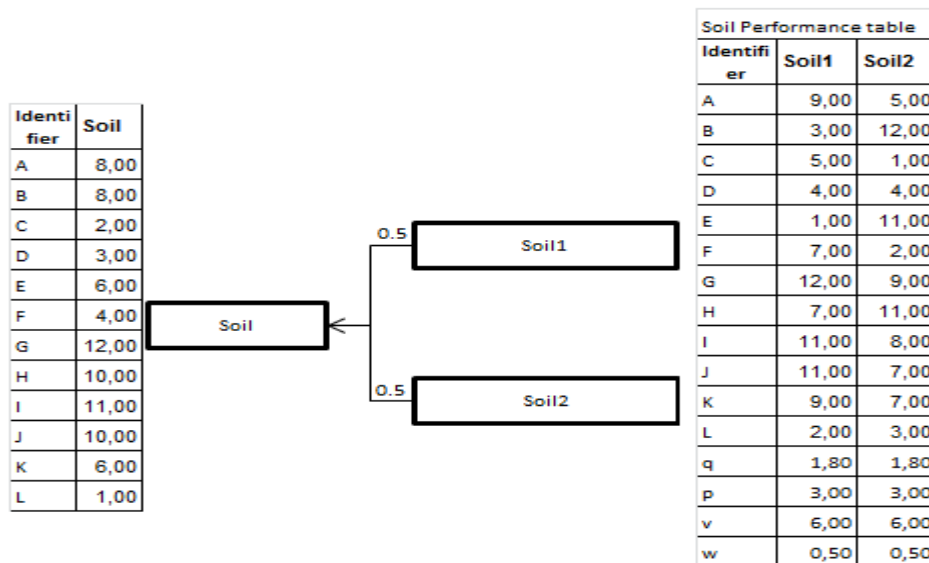


Figure 13 Ranking Soil criterion at first upper level

This performance values is then combined with the performance value obtained for Groundwater, figure 12, at the second upper level. The complete preorder obtained as a result of evaluating the alternatives on the two criteria; Soil and Groundwater results in the ranking of the alternative at the Groundwater node, figure 14. This ranking is transformed into scores which would be used as performance measure for the alternatives in relation to the *Environment* criterion at the highest level.

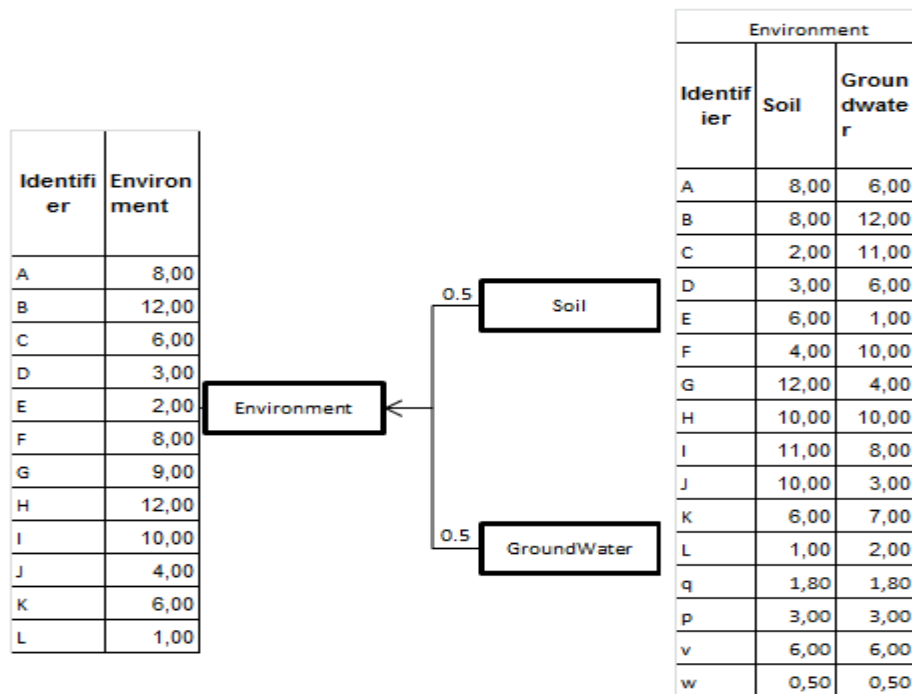


Figure 14 Environment criterion Ranking at highest level

5.1.5. Final Ranking at root level of the hierarchy

The process is completed when the aggregation is propagated through all the branches of the hierarchy for which a final complete preorder is obtained as a recommendation for the decision making process. In this case, the final ranking is produced with the application of the adapted ELECTRE III-H method to the evaluation of the alternatives on the two main criteria; Human Health Risk and Environment figure 15.

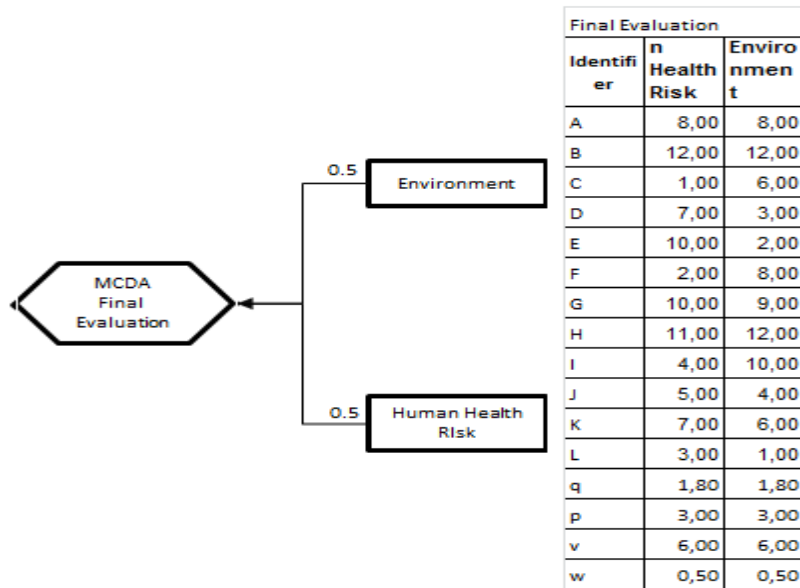


Figure 15 Final evaluation at root level

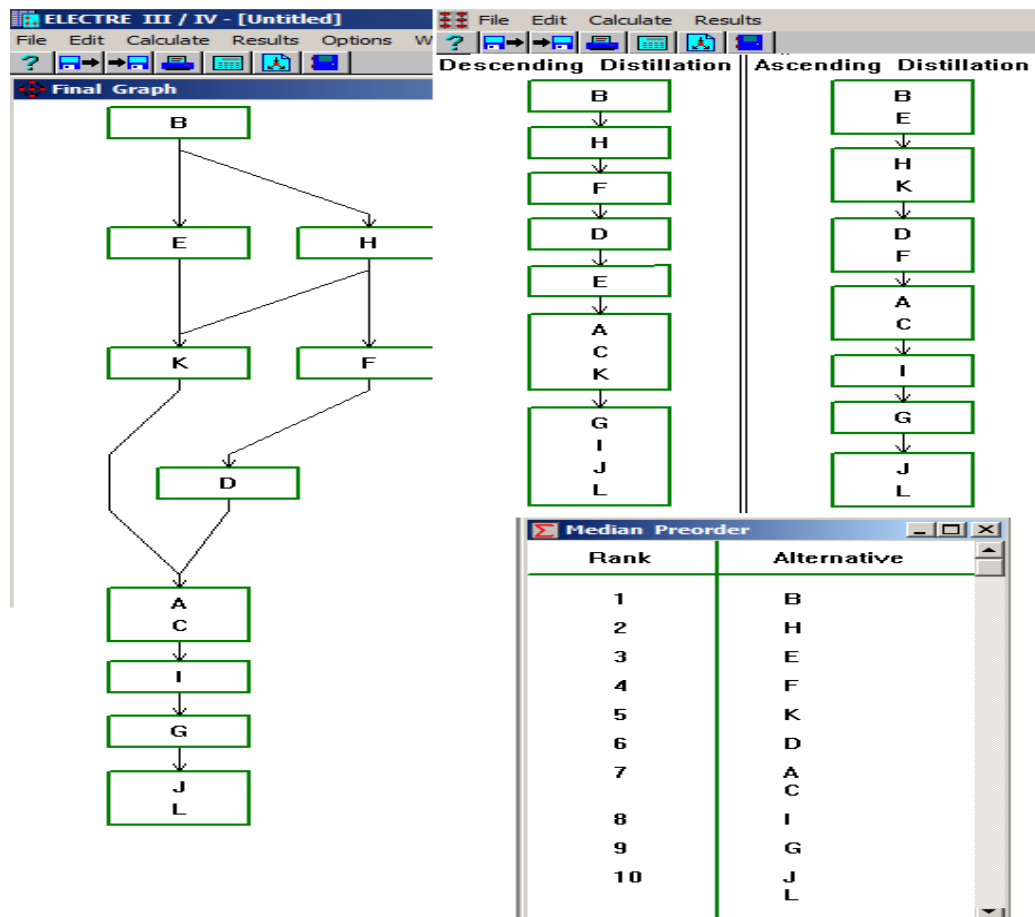


Figure 16 Final Ranking of sewage Soil disposal

The final complete preorder is obtained by applying the median preorder to the result of the downward and upward distillation. As a result, a final ranking is produced with alternative B being the best option preferable to all other alternatives, followed by alternative H which is classified as the second best option, L being the worst of all the alternatives. Figure 16 below displays the results of the final graph, downwards and upwards distillations and the median preorder obtained at the root level.

5.2. Application of version 2: Net Flow Score Method (NFS)

In this section, we applied the proposed NFS version to the data provided in table4 and thresholds in table5, with the aim of recommending the best alternative for the management of sewage sludge disposal which was defined on 12 alternatives and 12 hierarchically structured criteria. As in the application of the first version, we will split the hierarchy to several small parts with description of the process at each of the nodes of the hierarchy. To understand the process involved in the application of the NFS method, we will illustrate the process at the lowest level, evaluating the alternatives to produce a ranking at the node corresponding to the first upper level criterion, *Population*.

5.2.1. Ranking Human Health Risk criterion at the lowest level of the hierarchy

Given the data for the terminal criteria, *population type*, *population density* and *distance to urban areas*, with their respective weights and thresholds as represented in table4, table5 and figure 6, to obtain the ranking at the node corresponding to the population criterion on the hierarchy, we computed the *partial concordance index* $C_i(a, b)$ for each pair of alternatives (a, b) in terms of each of the three criteria according to equation3.15. These results are then combined, using equation3.16, to obtain the *global concordance index* represented in the table below.

Alternatives	A	B	C	D	E	F	G	H	I	J	K	L
A	1	1	1	0,6	0,6	0,6	0,2	0,2	0,2	0,4	0,4	0,4
B	1	1	1	0,6	0,6	0,6	0,2	0,2	0,2	0,4	0,4	0,4
C	1	1	1	0,6	0,6	0,6	0,2	0,2	0,2	0,4	0,4	0,4
D	0,47	0,47	0,47	1	1	1	0,6	0,6	0,6	0,4	0,4	0,4
E	0,47	0,47	0,47	1	1	1	0,6	0,6	0,6	0,4	0,4	0,4
F	1	0,47	0,47	1	1	1	0,6	0,6	0,6	0,4	0,4	0,4
G	0,87	0,87	0,87	1	1	1	1	1	1	0,4	0,4	0,4
H	0,87	0,87	0,87	1	1	1	1	1	1	0,4	0,4	0,4
I	0,87	0,87	0,87	1	1	1	1	1	1	0,4	0,4	0,4
J	0,6	0,6	0,6	1	1	1	0,6	0,6	0,6	1	1	1
K	0,6	0,6	0,6	1	1	1	0,6	0,6	0,6	1	1	1
L	0,6	0,6	0,6	1	1	1	0,6	0,6	0,6	1	1	1

Table 6 Concordance table for terminal criteria leading to population criterion

The next step is to calculate the *discordance index* $D_i(a, b)$ for all the alternatives in terms of each one of the decision criteria according to equation 3.17 and the *credibility index* for all the alternatives using equation 3.18. The credibility matrix is a measure of the strength of the claim that “alternative a is at least as good as alternative b ”. As mentioned earlier, the computation of the discordance index and the credibility index are combined to produce a single outranking matrix for the node, table 7. In the transformation from table 6 to table 7, a decrease is noted in some of the values for example, the strength of the concordance that alternative J is better than alternative A with a value of 6 (table 6: row 10 column 1) was reduced to 0,46 (table 7: row 10 column 1). This is due to the strong disagreement of the criterion population risk exercising the discordance in the computation of the credibility matrix using the power of the veto threshold. In cases where at least one of the criteria produce a discordance value equal to 1, the values corresponding to the concordance matrix will become zero when their credibility indices are computed. An example of this situation is produced in table 6 ((row 4 column 8) where the concordance matrix was reduced from 0.6 to 0 in the credibility matrix of table 7 ((row 4 column 8)).

Alternatives	A	B	C	D	E	F	G	H	I	J	K	L
A	1	1	1	0,6	0,6	0,6	0,07	0,07	0,07	0,13	0,13	0,13
B	1	1	1	0,6	0,6	0,6	0,07	0,07	0,07	0,13	0,13	0,13
C	1	1	1	0,6	0,6	0,6	0,07	0,07	0,07	0,13	0,13	0,13
D	0,45	0,45	0,45	1	1	1	0	0	0	0	0	0
E	0,45	0,45	0,45	1	1	1	0	0	0	0	0	0
F	0,97	0,45	0,45	1	1	1	0	0	0	0	0	0
G	0,87	0,87	0,87	1	1	1	1	1	1	0	0	0
H	0,87	0,87	0,87	1	1	1	1	1	1	0	0	0
I	0,87	0,87	0,87	1	1	1	1	1	1	0	0	0
J	0,46	0,46	0,46	1	1	1	0	0	0	1	1	1
K	0,46	0,46	0,46	1	1	1	0	0	0	1	1	1
L	0,46	0,46	0,46	1	1	1	0	0	0	1	1	1

Table 7 Outranking matrix for the generation of population criterion

The exploitation of the outranking relation is done with the NFS procedure instead of the distillation procedure used in the original ELECTRE III method. With the outranking relation of table7, we strengthen the degree of credibility by taking into account only those relations which enjoy relatively high credibility, that is to say, by defining a threshold $\delta = 0.5$ regulating the flow of the arcs S such that $S(a,b) > \delta$ signify that all entries of the outranking matrix greater than or equal to 0.5 will be automatically set to 1 while values with $S(a,b) < \delta$ will become 0, table8.

Alter	A	B	C	D	E	F	G	H	I	J	K	L
A	1	1	1	1	1	1	0	0	0	0	0	0
B	1	1	1	1	1	1	0	0	0	0	0	0
C	1	1	1	1	1	1	0	0	0	0	0	0
D	0	0	0	1	1	1	0	0	0	0	0	0
E	0	0	0	1	1	1	0	0	0	0	0	0
F	1	0	0	1	1	1	0	0	0	0	0	0
G	1	1	1	1	1	1	1	1	1	0	0	0
H	1	1	1	1	1	1	1	1	1	0	0	0
I	1	1	1	1	1	1	1	1	1	0	0	0
J	0	0	0	1	1	1	0	0	0	1	1	1
K	0	0	0	1	1	1	0	0	0	1	1	1
L	0	0	0	1	1	1	0	0	0	1	1	1

Table 8 Net flow score binary matrix

Next, we represent the resulting matrix as a directed graph, where nodes correspond to alternatives and arcs to outranking relations. An arc going from node a to node b would imply

that a outranks b (aSb), and the flow on this arc is equal to $S(a,b)$. In this case, arcs going from node a to other nodes will be represented by the row represented by the outranking relation of a with other alternatives in the outranking matrix while, arcs entering the node a from other nodes will be represented by the column of the credibility matrix corresponding to a .

The complete preorder (ranking) of alternatives is produced from the graph using equation 3.19 of the Net Flow Score procedure as follows:

$$NFS(a_i) = \sum_{j=1}^m a_{ij} - \sum_{j=1}^m a_{ji} \text{ where } i = (1, \dots, m) \text{ and } a_i \in A \text{ is an alternative}$$

Where $\sum_{j=1}^m a_{ij}$ is the sum of the rows of the matrix corresponding to alternative a in the credibility matrix and $\sum_{j=1}^m a_{ji}$ is the sum of the columns of the matrix corresponding to alternative a .

The sum of the row of the matrix corresponding to alternative a_i is represented by $\sum_{j=1}^m a_{ij}$, while $\sum_{j=1}^m a_{ji}$ is the sum of the column of the alternative. Next, we computed the position of each of the alternatives using the balance of flows of the arcs entering and leaving a node. These positions are then used to rank the alternatives from best to worst according to their NFS, table 9.

5.2.2. Ranking Human Health Risk criterion at upper levels

The net flow score obtained for each of the alternatives is used to sort the alternatives into order where the alternative with the highest NFS is regarded as the best alternative and it will be ranked higher than any other alternatives in the set of alternatives whereas, the alternative with the least NFS will be ranked at the bottom of the complete order produced. In this case, alternatives G , H and I are ranked higher than all the other alternatives while alternatives D and E are the least preferred alternatives.

Alternatives	Position		Alternatives best to worst	Sort by Position	Scores inherited by population
A	-1		G	6	12
B	0		H	6	12
C	0		I	6	12
D	-9		J	3	9
E	-9		K	3	9
F	-8		L	3	9
G	6		B	0	6
H	6		C	0	6
I	6		A	-1	4
J	3		F	-8	3
K	3		D	-9	2
L	3		E	-9	2

Table 9 NFS ranking for population criterion at the lowest level

Once the population criterion has been evaluated, this process is repeated with the terminal criteria corresponding to Landscape node to obtain the ranking of the alternatives. The results at the two nodes are then combined to produce the performance table, table 10, which would lead to the ranking at the second upper level node belonging to *population risk* criterion.

Identifier	Population	Landscape		Identifier	Population Risk
A	4	12		A	7,00
B	6	12		B	12,00
C	6	12		C	12,00
D	2	9		D	5,00
E	2	9		E	5,00
F	3	9		F	6,00
G	12	6		G	12,00
H	12	6		H	12,00
I	12	6		I	12,00
J	9	3		J	5,00
K	9	3		K	5,00
L	9	3		L	5,00
q	1,8	1,8			
p	3	3			
v	6	6			
w	0,5	0,5			

Table 10 Performance table and rank scores for population risk criterion

Likewise the ranking produce for population risk is combined with that of the terminal criterion *Labor Risk* to generate another ranking for the criterion *exposure risk* as shown in table11.

Identifier	Popula tion Risk	Labor Risk
A	7	0,4
B	12	0,6
C	12	0,2
D	5	0,8
E	5	0,97
F	6	0,6
G	12	0,4
H	12	0,6
I	12	0,2
J	5	0,8
K	5	0,97
L	5	0,6
q	1,8	0,2
p	3	0,3
v	6	0,5
w	0,8	0,2



Identifier	Exposure Risk
A	1,00
B	12,00
C	9,00
D	9,00
E	9,00
F	3,00
G	10,00
H	12,00
I	9,00
J	9,00
K	9,00
L	3,00

Table 11 Performance table and rank score for ingestion dose

The scores obtained by evaluating the alternatives on their performance values of exposure risk and the *ingestion dose* terminal criterion yield the ranking at the human health risk highest level node table12, with alternative *E* rated better than all other alternatives and alternatives *F* and *L* the least preferred.

Identifier	Exposure Risk	Ingestion Dose	Alternatives	Human Health Risk
A	1	0,63	E	12,00
B	12	0,7	B	11,00
C	9	0,52	H	11,00
D	9	0,7	D	9,00
E	9	0,83	G	8,00
F	3	0,5	K	7,00
G	10	0,63	C	6,00
H	12	0,7	I	6,00
I	9	0,52	J	6,00
J	9	0,52	A	3,00
K	9	0,6	F	2,00
L	3	0,5	L	2,00
q	1,8	0,05		
p	3	0,1		
v	6	0,2		
w	0,6	0,4		

Table 12 Human health risk criterion performance table

This result is later combined with the ranking obtained for the other channel of the branch corresponding to the Environment node to obtain a final ranking.

5.2.3. Ranking Environment criterion at lower levels

The NFS procedure for ranking along the branch of the Environment criterion is applied in the same manner as in the human health risk branch. In this case, three terminal criteria are combined to produce a ranking for criterion Soil1. Likewise, two terminal criteria are used to evaluate alternatives to generate a ranking at the node Soil2. These rankings obtained at the nodes for Soil1 and Soil2 are used as performance values, table13, on the two criteria at the first upper level of this branch.

Identifier	Soil1	Soil2
A	7	4
B	2	12
C	9	4
D	4	7
E	1	12
F	9	4
G	11	8
H	6	12
I	12	9
J	11	4
K	5	7
L	3	7
q	1,8	1,8
p	3	3
v	6	6
w	0,5	0,5



Alternatives	Soil
A	3
B	9
C	6
D	3
E	8
F	4
G	12
H	10
I	12
J	7
K	6
L	1

Table 13 Soil performance table and rank scores for environment criterion branch

Similarly, the ranking at the groundwater node is generated by applying the NFS method to the two terminal criteria *Gw_contamination* and *Gw_vulnerability*. Scores are allocated in relation to the position of the alternatives to generate their performance value on the criterion *groundwater* as shown in table14.

Identifier	GW_V ULNER	GW_Con taminati on
A	0,4	0,6389
B	0,5	0,6
C	0,5	0,55
D	0,4	0,7106
E	0,5	0,3667
F	0,5	0,54
G	0,4	0,6389
H	0,5	0,6
I	0,5	0,55
J	0,4	0,622
K	0,5	0,475
L	0,5	0,38
q	0	0,08
p	0,09	0,15
v	0,3	0,3
w	0,5	0,5



Alternat ives	Ground water
A	5,00
B	12,00
C	10,00
D	6,00
E	5,00
F	10,00
G	5,00
H	12,00
I	10,00
J	1,00
K	7,00
L	5,00

Table 14 Groundwater performance table and rank scores

5.2.4. Ranking Environment criterion at upper levels

The NFS method application commence with the evaluation of the alternatives on the criteria Soil1 and Soil2 whose scores have been generated from ranking propagated by lowest level nodes. This leads to the generation of a ranking of alternatives which are translated to the scores measuring the performance of the alternatives on the criterion Soil at the second upper level node. Next, the performance of Soil and Groundwater, table15, are aggregated with their respective weights and thresholds, using the NFS method to yield the scores on the Environment criterion.

Alternatives	Soil	Groundwater
A	3	5
B	9	12
C	6	10
D	3	6
E	8	5
F	4	10
G	12	5
H	10	12
I	12	10
J	7	1
K	6	7
L	1	5
q	1,8	1,8
p	3	3
v	6	6
w	0,5	0,5



Alternatives	Environment
A	4
B	10
C	8
D	2
E	6
F	7
G	9
H	11
I	12
J	4
K	5
L	2

Table 15 Performance table and rank scores for environment criterion

5.2.5. Final evaluation at root node of the hierarchy

The NFS method is applied at the root node to obtain the final ranking of the alternatives by evaluating their performance on the two criteria, *human health risk* and *environment* at the highest level. Recall that the scores of these two criteria have been inherited from the propagation of rankings from the terminal criterion through the intermediate nodes up to this level. Table16 displays the result of the final recommendation with the NFS method.

This method recommends that alternatives *B* and *H* have equal importance at the top of the ranking as the two most preferred alternatives while *L* is recommended as the worse alternative.

Alternatives	Human Health	Environment
A	3	4
B	11	10
C	6	8
D	9	2
E	12	6
F	2	7
G	8	9
H	11	11
I	6	12
J	6	4
K	7	5
L	2	2
q	1,8	1,8
p	3	3
v	6	6
w	0,5	0,5



Alternatives	Final Ranking
B	12,00
H	12,00
E	10,00
I	10,00
G	8,00
C	7,00
D	6,00
K	6,00
J	4,00
F	3,00
A	2,00
L	1,00

Table 16 Final ranking with the NFS method

5.3. Application of version3: Credibility propagation (CRED) method

In this section we applied the proposed CRED method to select the best for 12 alternatives evaluated on 12 criteria for the management of sewage sludge disposal. Given the hierarchical structure of criteria, figure6, and the data for sewage sludge disposals, table 4, we aim to rank the alternative from best to worst, commencing with the application of the original ELECTRE III method formulation to obtain credibility matrices at the lowest level and then propagate the credibility matrices to upper levels of the hierarchy with only the weights of the intermediate level criteria. This way, the final ranking is obtained at the root node with the credibility matrix of the node at the highest level.

5.3.1. CRED method at the lowest level

The application of the CRED method to the terminal criteria at the lowest level of the human health risk criterion yields credibility matrices inherited at the first upper level by each of the criteria, *Population* and *Landscape*.

Alternatives	A	B	C	D	E	F	G	H	I	J	K	L
A	1	1	1	0,6	0,6	0,6	0,07	0,07	0,07	0,13	0,13	0,13
B	1	1	1	0,6	0,6	0,6	0,07	0,07	0,07	0,13	0,13	0,13
C	1	1	1	0,6	0,6	0,6	0,07	0,07	0,07	0,13	0,13	0,13
D	0,45	0,45	0,45	1	1	1	0	0	0	0	0	0
E	0,45	0,45	0,45	1	1	1	0	0	0	0	0	0
F	0,97	0,45	0,45	1	1	1	0	0	0	0	0	0
G	0,87	0,87	0,87	1	1	1	1	1	1	0	0	0
H	0,87	0,87	0,87	1	1	1	1	1	1	0	0	0
I	0,87	0,87	0,87	1	1	1	1	1	1	0	0	0
J	0,46	0,46	0,46	1	1	1	0	0	0	1	1	1
K	0,46	0,46	0,46	1	1	1	0	0	0	1	1	1
L	0,46	0,46	0,46	1	1	1	0	0	0	1	1	1

Table 17 Credibility matrix inherited by population criterion

Likewise, credibility matrices are propagated to Soil1, soil2 and Groundwater criterion from the terminal criteria of the environment criterion. These matrices are obtained following the original ELECTRE III method for computing credibility index, whereby the concordance and discordance indices are used to produce values for the credibility matrix.

Alternatives	A	B	C	D	E	F	G	H	I	J	K	L
A	1	1	1	1	1	1	0,76	0,76	0,76	0,72	0,72	0,72
B	1	1	1	1	1	1	0,76	0,76	0,76	0,72	0,72	0,72
C	1	1	1	1	1	1	0,76	0,76	0,76	0,72	0,72	0,72
D	0,7	0,7	0,7	1	1	1	0,71	0,71	0,71	0,68	0,68	0,68
E	0,7	0,7	0,7	1	1	1	0,71	0,71	0,71	0,68	0,68	0,68
F	1	0,7	0,7	1	1	1	0,71	0,71	0,71	0,68	0,68	0,68
G	0,13	0,13	0,13	0,55	0,55	0,55	1	1	1	1	1	1
H	0,13	0,13	0,13	0,55	0,55	0,55	1	1	1	1	1	1
I	0,13	0,13	0,13	0,55	0,55	0,55	1	1	1	1	1	1
J	0	0	0	0	0	0	0,35	0,35	0,35	1	1	1
K	0	0	0	0	0	0	0,35	0,35	0,35	1	1	1
L	0	0	0	0	0	0	0,35	0,35	0,35	1	1	1

Table 18 Credibility matrix inherited by landscape criterion

5.3.2. CRED method application at upper levels

At this level, to avoid the weights of the criterion being used as a substitution rate, we defined another credibility function $S_j^U(a, b)$ and a threshold $\hat{S} \geq 0.5$ at each intermediate node such that, any value in the inherited credibility matrix ($S^L(a, b)$) greater than or equal to $\hat{S}$ will get a score of 1 while values less than $\hat{S}$ will get a score of 0. This function and threshold are applied to the inherited matrices and the weights of the criteria to generate a new matrix for the criterion Population Risk (at second upper level) corresponding to the branch of human health risk.

At the second upper level, the credibility matrix for exposure Risk is computed in a similar manner to that of the population risk except that the second criterion labor risk is a terminal criterion, this implies that to aggregate the credibility matrix of labor risk, we first needed to calculate a credibility index for the labor Risk criterion. Then, we combine the credibility matrix obtained with that of the population risk to generate a credibility index for exposure risk at the second upper level using the CRED aggregation method. Next, the credibility matrix of exposure risk is combined with another terminal criterion to produce a credibility matrix at the human health risk highest level node.

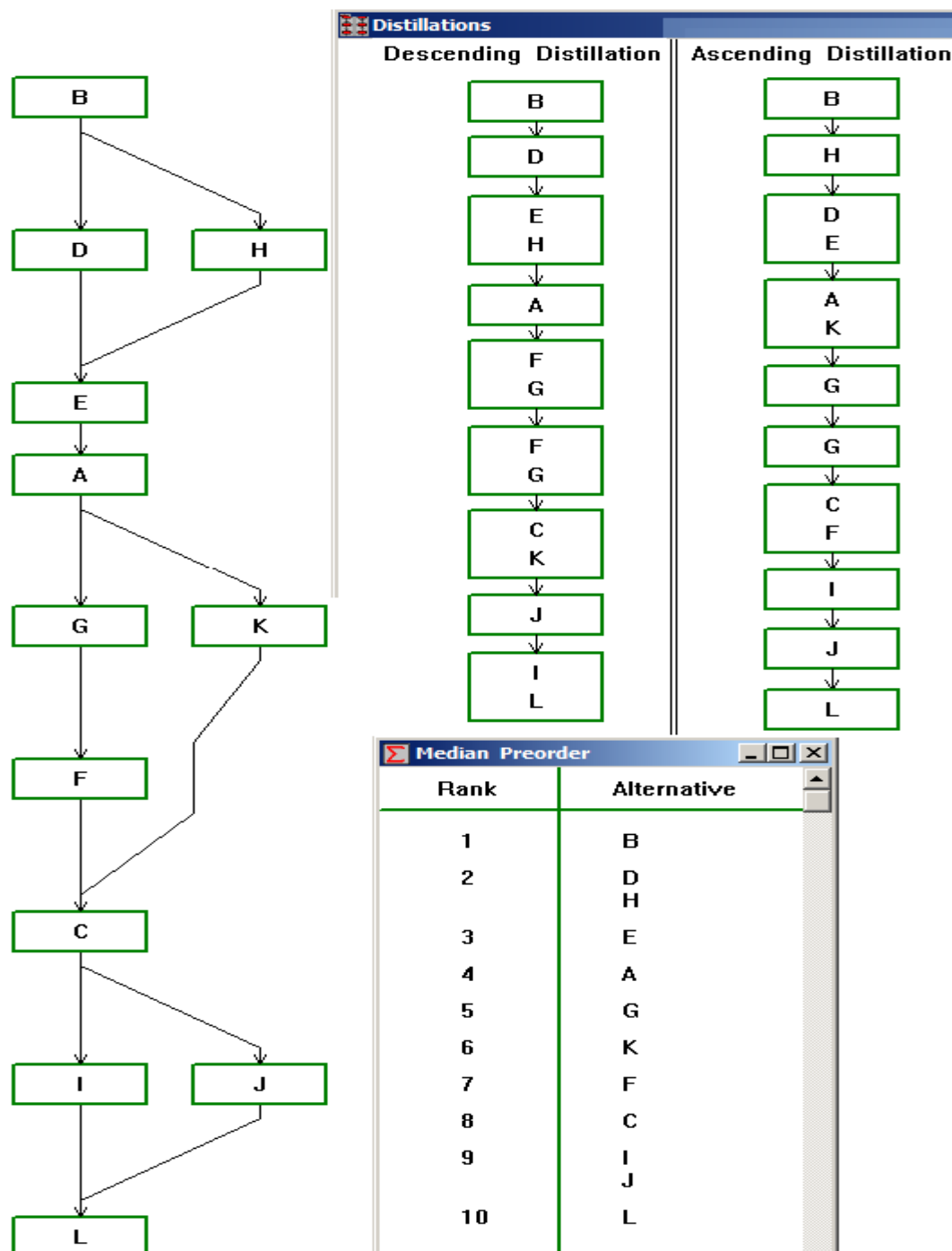


Figure 16 Final graph, distillations and median preorder at root node

On the channel of the environment criterion, the CRED method is applied in a similar manner to the terminal criteria to obtain the credibility matrix which is propagated to Soil1, Soil2 and Groundwater at the first upper level. These matrices are then combined using the threshold and the CRED function to yield a matrix at the environment node.

At the highest level, the process is completed when the aggregation is propagated through all the branches of the hierarchy for which a final credibility matrix is obtained at the root node by a combination of the credibility matrices of the two highest level criteria, human health risk and environment. The final ranking is produced by exploiting the combination of the partial order generated by the downwards and upwards distillation with the median preorder procedure. As shown in figure16, this version recommends alternative B as the best option followed by alternatives D and H which are classified as incomparable but preferred to all other alternatives except B, L being the worst of all the alternatives. Figure 16 displays the final ranking obtained with the final graph, distillations and median preorder at the root level.

5.5. Summary of results of the three methods

The result summary of table19 shows the ranking recommended by each of the methods. It can be concluded that each of the methods clearly selected alternative B as the best alternative except for the version 2 where B shares the best position with H. For the intermediate level positions, some noticeable rank reversals have been observed especially when the rankings produced by version 2 is compared to those of versions 1 and 3 for alternatives A and C for example. In this case, alternative A was given scores 10 and 8 by versions 1 and 3 respectively while it got a score of 2 from version 2. This led to a series of further investigation which as indicated in the result of the sensibility analysis, alternative A tend to get a low score from version2 irrespective of the variations in the threshold parameters. For the worst alternative, all the three method selected alternative L as the least preferred of all the alternatives

Alternatives	V1	V2	V3
A	10	2	8
B	12	12	12
C	2	7	4
D	5	6	11
E	6	10	9
F	3	3	5
G	10	8	7
H	11	12	11
I	8	10	3
J	5	4	3
K	8	6	7
L	1	1	1

V1	V2	V3
B	B H	B
H	E I	D H
A G	G	E
I K	C	A
E	D K	G K
D J	J	F
F	F	C
C	A	I J
L	L	L

Table 19 Final ranking produced by the three methods

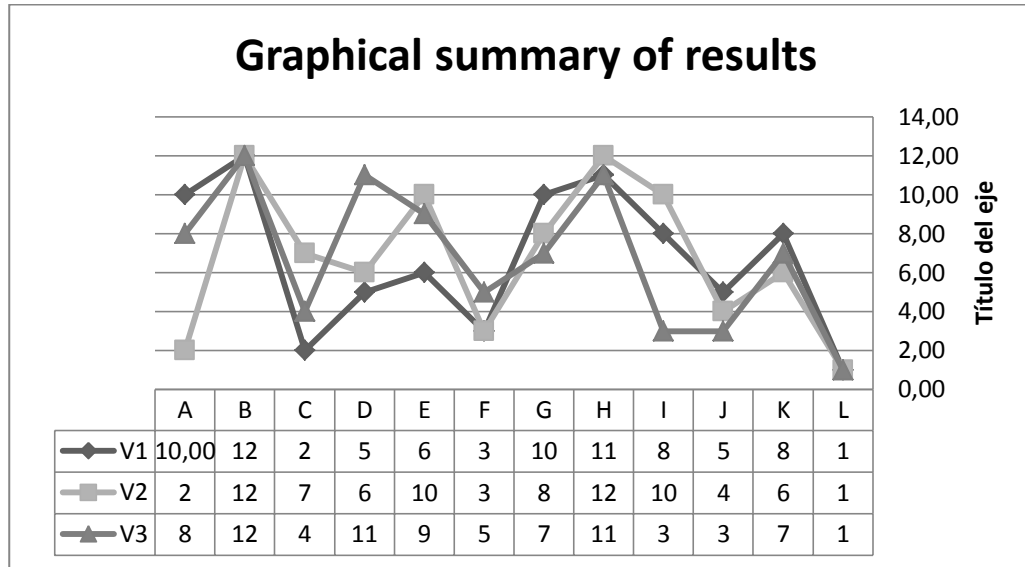


Figure 17 Plot of final ranking

5.6. Sensitive analysis

In decision making analysis, the analyst often choose values which may not be very well known for the parameters, this could be due to information being incomplete or rather reliable. Technical parameters such as weights, veto thresholds, indifference and preference thresholds, concordance levels to mention a few are very important to build decision models. For this reason, it is important to examine the impact of changes in the values of the parameters used to define the outranking relation.

For the test carried out in section 4.4, central values have been chosen for each of the parameters in order to obtain a first solution. In this section, we performed a sensitivity analysis to study the influence of the threshold parameters on the solutions, to enable us to determine the sensitivity of the methods for the different plausible set of values. This way, we can have some guarantee that the decision proposed by the methods will not lead to a catastrophe when different values are used for the parameters in the test.

As would have been observed in section 4, considering 12 alternatives for the test in this thesis imply that the volume of computation required for the sensitivity analysis will augment with respect to the number of sensitivity test or times we vary the parameter values.

5.6.1. Sensitivity with respect to the thresholds

When the ELECTRE methods are used to make recommendations in decision making problems, the selection of the most preferred alternative depend partly in the thresholds specified for the criteria. Since the determination of precise threshold values is not an easy task, it is appropriate to justify the decision by showing that it is insensitive to changes in the values of these parameters [64].

The variations of the parameters for the analysis are conducted as outlined in the table below.

a) Original version

Terminal criteria

Thresh olds	Temper ature	Precipit ation	Crop type	Popul ation type	Pop ulation den sity	Distan ce to urban areas	GW conta mination	GW vulnera bility	Inges tion dose	Labo r	Biod iversity	Nutri ents	OM	pH	Soil cont amin ation
q	1.00	20.00	0.15	0.00	0.10	1.00	0.08	0.00	0.05	0.20	0.05	0.02	0.01	0.00	0.08
p	1.50	30.00	0.35	1.00	0.30	4.00	0.15	0.09	0.10	0.30	0.10	0.05	0.12	0.12	0.10
v	5.00	500.00	0.50	2.00	0.60	10.00	0.30	0.30	0.20	0.50	0.20	0.15	0.20	0.15	0.25

Table 20 Original values of terminal criteria used in the test

Intermediate criteria

Threshold s	Human health risk	Environme nt	Exposur e risk	Populati on risk	Popul ation	Landscap e	Soil	Groundwa ter	Soil1	Soil2
q	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
p	3	3	3	3	3	3	3	3	3	3
v	6	6	6	6	6	6	6	6	6	6

Table 21 Original values of intermediate level criteria used in the test

- b) First sensitivity test, varying terminal criteria: The suitability criteria in the set of criteria have been varied to analyse their influence on the solution. The reason for this decision is to study how strict restrictions on the thresholds parameters of these attributes that are important to human health and environment affect the final recommendations.

Thresholds	Temperature	Precipitation	Crop type	Population type	Population density	Distance to urban areas	GW contamination	GW vulnerability	Ingestion dose	Labor	Biodiversity	Nutrients	OM	pH	Soil contamination
q	1.00	20.00	0.15	0.00	0.10	1.00	0.05	0.00	0.00	0.05	0.00	0.00	0.05	0.05	0.05
p	1.50	30.00	0.35	1.00	0.30	4.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
v	5.00	500.00	0.50	2.00	0.60	10.00	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30

Table 22 Values for first sensitivity test

- c) Second sensitivity test, varying intermediate level criteria: the objective of this test is to analyse the influence of the variation of the threshold values of the intermediate level criteria

Thresholds	Human health risk	Environment	Exposure risk	Population risk	Population	Landscape	Soil	Groundwater	Soil1	Soil2
q	0	0	0	0	0	0	0	0	0	0
p	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
v	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2

Table 23 Values for second sensitivity test

5.6.2. Results of sensitivity analysis

For simplicity, we present the final results of the sensitivity analysis in tabula and graphical forms for each of the proposed methods. The scores corresponding to each of the 12 alternatives are recorded and compared for each test. The alternative with a score of 12 will be considered better than all the other alternatives while that with a score of 1 will be ranked as the worst of all the alternatives.

The result as shown in figure 18 shows that the ranking is very stable with respect to the ranks of the alternatives in the three tests. This means that the positions of the best and worst alternatives are clearly established and varying the values of the thresholds has no significant influence on the process of selecting the best alternative. In the original version, the alternatives *B* and *H* are ranked equally preferred to all other alternatives while the two tests for sensitivity analysis coincide that *B* is better than all the others including *H* which is ranked as second best. The alternative *G* is clearly ranked third in the three tests though, it is ranked a scale higher in the original test than in the other tests.

5.6.2.1. Sensitivity analysis for version1

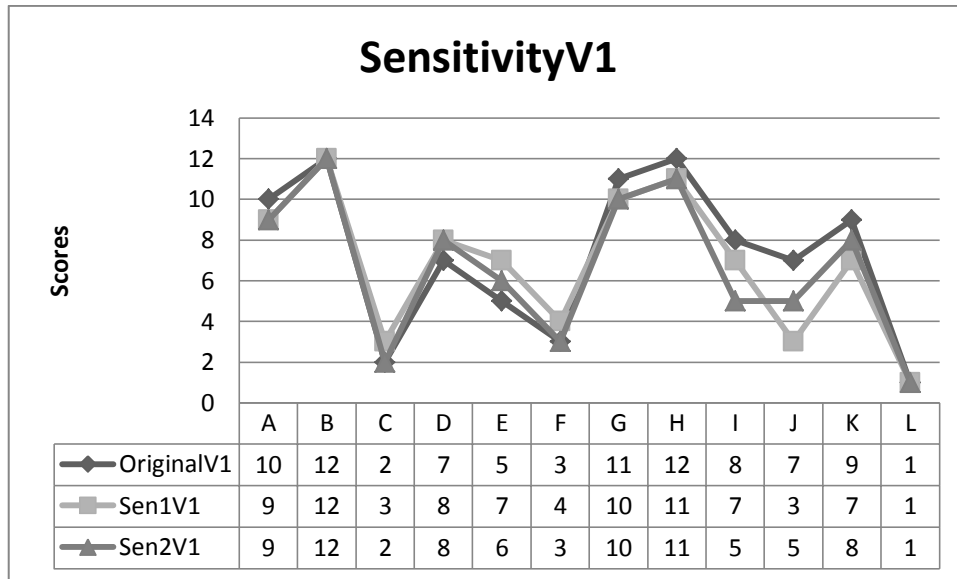


Figure 18 Sensitivity analysis for ELECTRE III - H with median preorder: Version1

With respect to the intermediate positions, some changes can be observed with the variation of the parameter values. For example, alternative *J* occupying the 7th position in the original test was relegated to the 3rd position when some threshold values of the terminal criteria were varied and to the 5th position with variation of the intermediate parameter values. For the least preferred criterion, there is a unanimous recommendation in the selection of the three tests where alternative *L* ranks as the worst alternative.

Regardless of the changes found in the intermediate positions, *B* is clearly the alternative with the best combination of all, followed by alternative *H*. This analysis shows that this method is not very sensitive to changes in the variation of the threshold values and it can be applied in decision problems with hierarchical structure of criteria.

5.6.2.2. Sensitivity analysis for version2

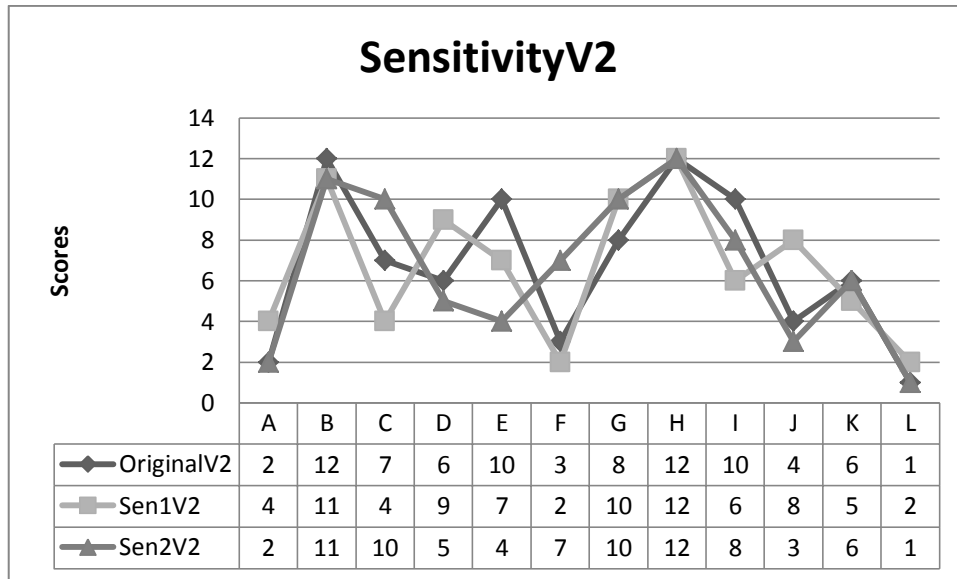


Figure 19 Sensitivity analysis for NFS method

The sensitivity analysis of the second version shows slightly more variation than those of the first version. Though, it coincides with the recommendation of the original test of version1 that alternatives *B* and *H* are equally preferred to all other alternatives with alternative *L* sharing the position of the worst version with alternative *F* for the 1st sensitivity and being the worst version for the original test and the second sensitivity test. A high fluctuation can be observed in the intermediate positions where alternatives *C*, *D*, *E*, *F*, *I* and *J* tend to alternate positions with different variation in the parameter values. This indicates that the changes in the values of the thresholds have an influence on these alternatives which consequently produce the changes in the ranking. Though the recommendations for the selection process clearly recommends *H* is slightly better *B* which is selected as the second best alternative, it would be worth studying how the performance of the alternatives in the intermediate positions have been affected by the criteria whose parameters were varied.

5.6.2.3. Sensitivity analysis for version3

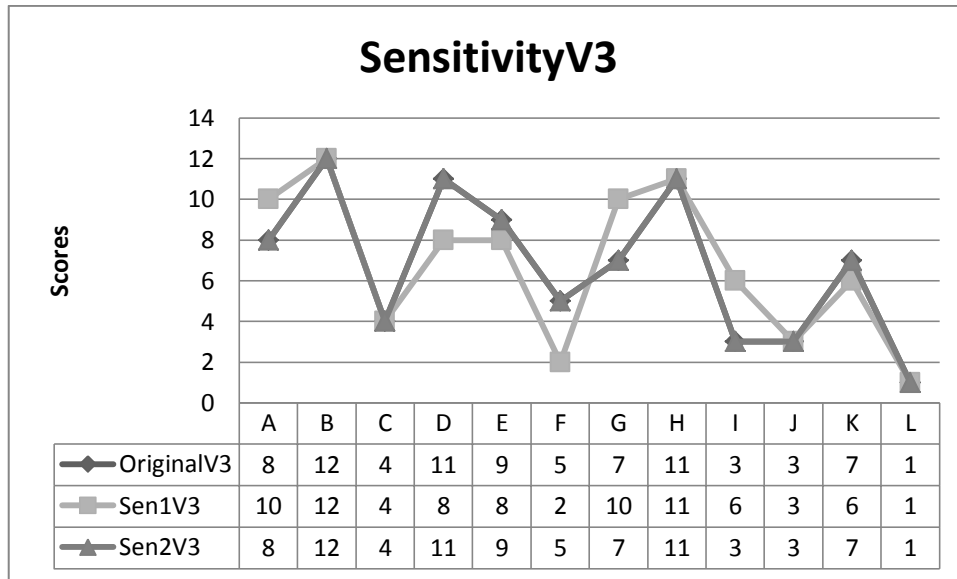


Figure 20 Sensitivity analysis for CRED method

In this case, it can be observed that the best, second best and last positions are clearly defined with alternative B occupying the best position, followed by alternative H and alternative L being the worst alternative. A close observation of the result of the original test and that of the 2nd sensitivity test shows that the two tests result in practically the same ranking for all the alternatives. This is simply due to the fact that version3 does not require the use of threshold values for intermediate level criteria thus; changes in the values of these parameters will have no influence whatsoever on the selection process.

For this method, positions such as those for alternatives B, H, C, J and L are clearly defined with slight variations of 1 scale point for alternatives E and K while A fluctuated by 2 scale points and D, F, G and I experienced rank reversals of 3 scale points.

6. Conclusions and future work

In this thesis, we have proposed three methods which aim at extending the ELECTRE III approach to decision making problems with hierarchically structured criteria. For this purpose, the following goals were formulated, all of which have been completely accomplished.

1. The ELECTRE methodology has been studied extensively with special dedication to the ELECTRE III method which served as a base for developing the proposed methods.
2. We extended the concepts of concordance and non-discordance tests to criteria with ordinal scales and partial orders of alternatives evaluated by these criteria.
3. Based on the hypothesis, theories and approaches extracted from 1 and 2 above, we proposed three adaptations of the ELECTRE III method to deal with hierarchical structure of criteria. The first proposed method called the *ELECTRE III - H with median preorder* is a natural adaptation of the ELECTRE methods in that, it conserves all the usual procedures of ELECTRE III at all levels of the hierarchy. The second method, referred to as the *NFS method*, utilise some properties of the ELECTRE III method to obtain outranking relations for the alternatives. This relation is further exploited using the Net flow score procedure instead of the distillation procedure of the original ELECTRE III method. The third procedure called the *CRED method* tends to simplify the process of data-passing up the hierarchy by propagating the credibility matrices from lower levels through the intermediate levels up to the root node. This way, the ranking of the alternatives is carried out only at the root node. All in all, the results obtained for the three methods gave some degree of confidence for their exploitation. It will be worth mentioning that the most reliable of them all is the ELECTRE III - H with median preorder method which would require a well-structured formulation of the threshold values at the intermediate levels of the hierarchy and some profound sensibility analysis in order to validate the efficiency of the method. In the case of the NFS and CRED methods, further development work would be required for the structuring and definition of the two approaches in agreement with the principles guiding ELECTRE methods.

4. To facilitate the implementation of the proposed methods, a Visual Basic application within Microsoft Excel spreadsheet was developed in conjunction with the Diviz software by Decision deck and the ELECTRE III/IV software courtesy of Prof. Roman Slowinski from the Institute of Computing Science, Poznan University of Technology, Poznan (Poland).
5. Finally, we applied the proposed methods to the problem of management of the disposal of sewage sludge generated during the water cleaning process in wastewater treatment plants. This is a multiple criteria ranking problem with a set of criteria organized into a hierarchical structure concerning two main aspects: impact for humans, and impact on the environment and ecosystems[62, 63]. In addition, we performed sensitivity analysis where the thresholds were varied for both terminal and intermediate criteria. In this case our attention is restricted mainly to the analysis of how the best alternatives recommended by each version are influenced by changes in the values of the parameters. This sensitivity analysis was particularly helpful in that it provides different scenarios and possibilities where the rankings of the alternatives can be used to better assess the different methods in terms of their sensitivity to variations in the parameter values. In summary, each of the three methods produced similar recommendation with regards to the best alternatives. The best and worst alternatives were not greatly influenced by the fluctuations in the parameter values though, intermediate criteria tends to experience reversal order in almost all the cases. This case would require further studies. Though, it is said to be common in outranking methods where the alterations of the parameters influence to some considerable extent the ranking of the alternatives in question.

With the research work carried out in this thesis, the following conclusions can be extracted from the proposed methodologies.

- The successful application of the three methods to the management of sewage sludge disposals demonstrates that ELECTRE methods can be applied to select the best alternatives in decision problems with hierarchically structured criteria. With sound theoretical framework structure, these methods can be applied to support decision makers in facing complex real-world decision problems that often involve a plethora of factors and criteria. Their potentials can be explored to make decisions in many fields, including artificial intelligence. These kinds of methods are is particularly interesting to develop new user-centered applications, in which, personal preferences are taken into

account to make; intelligent recommendations for ranking or choice decisions, content filtering or even for designing personalised interfaces.

- The many number of assumptions made regarding parameters and factors, specific to ELECTRE methods, made the design of the models quite complex. These factors and parameters require particular importance when the outranking relations are extended to produce explicit preference ordering at the lowest level and propagated to higher levels of the hierarchy. Taking all these into consideration, we have been able to produce three different independent methods instead of the single extension ELECTRE-H method requested in the thesis proposal.
- The primary appeal of the three methods is in the avoidance of the overly restrictive assumptions of values. This means that they provide the opportunity to make use of a rich array of preference models with concept such as incomparability and degree of preference which forms part of the decision models. This concepts help to channel the utility of the methods in the direction required by the decision maker concerning whether or not one alternative should be judged as more preferred than another when they are evaluated upon by hierarchically structured criteria.

As recommendations for future research work, some of the following areas can be of considerable interest.

- The design of the model whereby the concordance and the discordance level is propagated from the lowest level of the hierarchy through the intermediate nodes to the highest level where the outranking relation would be generated and exploited to obtain a final ranking. Furthermore, it would be of importance to formulate some structured definition for the determination of the thresholds for intermediate level criteria.
- A deeper analysis of the three methods would need to be completed in consultation with the sewage sludge experts to study how the impact of weight changes will affect the selection of the best alternatives with special attention to the ranking obtained at the intermediate levels. The possibility of including extra nodes to the models in order to value the performance of the methods could be a key factor in

building reliable ELECTRE models to solve decision problems with criteria hierarchy.

- The algorithm for computing the median preorder could be revised, since there may be different possibilities of dealing with the partial rankings obtained after the distillation procedure. In fact, the software Diviz and the software ELECTREIII/IV (developed in the group LAMSADE) gave different solutions. This step is crucial in the first method proposed. This raised a question on the possibility of an indebt study of the median order as in the original ELECTRE methods so that the proposed methods can be developed into a single complete system to be applied when solving problems involving hierarchical structure of criteria.

Bibliography

- [1] J. Figuera, *et al.*, *Multiple criteria decision analysis: State of the Art Surveys*, 2005.
- [2] J. Figueira, *et al.*, "ELECTRE methods: Main Features and Recent Developments," 2010.
- [3] B. Roy, *The outranking approach and the foundations of ELECTRE methods. Theory and Decision* vol. 31, 1991.
- [4] C. Arondel and P. Girando, "Sorting cropping systems on the basis of impact on groundwater quality," *European J. of Operational Research*, vol. 127, pp. 467-482, 2000.
- [5] O. Barda, *et al.*, "Multicriteria location of thermal power plants," *European J. of Operational Research*, pp. 332-346, 1990.
- [6] M. Gershon, *et al.*, "Multiobjective river basin planning with qualitative criteria," *Water Resources Research* vol. 18, pp. 193-202, 1982.
- [7] A. Dimitras, *et al.*, "Assessing financial risks using a multicriteria sorting procedure: The case of country risk assessment," *Foundations of Computing and Decision Science* vol. 29, pp. 97-109, 2001.
- [8] V. Gabrel, "Méthodology pour la planification de production de système d'observation de la terre par satellites: Aspects algorithmiques et multicritères.," Université Paris-Dauphine, 1994.
- [9] G. Colson, "The OR's prize winner and the software ARGOS: How a multijudge and a multicriteria ranking GDSS helps a jury to attribute a scientific award " *Computers & Operations Research*, vol. 27, pp. 741-755, 2000.
- [10] Y. Bétolaud and R. Février, "Conservation des forêts suburbaines et passage des autoroutes.," *Revue Forestière Française*, pp. 197-200, 1973.
- [11] F. Hallot, *et al.*, "Decision Support Methodologies for Acquisition of Military Equipment," *Systems Analysis and Studies Panel (SAS) Specialists*, p. 176, 2009.

- [12] B. Roy and R. Słowiński, "Handling effects of reinforced preference and counter-veto in credibility of outranking," *European J. Operational Research*, vol. 188, pp. 185-190, 2008.
- [13] M. Wiecek, *et al.*, "Multiple criteria decision making for engineering," *Omega*, vol. 36, pp. 337–339, 2008.
- [14] R. L. Keeney and H. Raiffa, *Decisions with Multiple Objectives, Preferences and Value Tradeoffs*. New-York: Wiley, 1976.
- [15] B. Roy and D. Bouyssou, "Procédures d'agrégation multicritère conduisant à un critère unique de synthèse," Working Paper, LAMSADE, Université de Paris-Dauphine, 1987.
- [16] P. Vincke, *MultiCriteria Decision-aid*: Wiley, 1992.
- [17] L. De Boer, *et al.*, "Outranking methods in support of supplier selection," *Journal of Purchasing and Supply Management*, vol. 4, pp. 109 -118, 1998.
- [18] E. Takeda, "A method for multiple pseudo-criteria decision problems," *Computers & Operations Research*, vol. vol. 28, pp. 1427 -1439, 2001.
- [19] J. Leyva-Lopez Carlos and E. Fernandez-Gonzalez, "A new method for group decision support based on ELECTREIII methodology," *Journal of Operational Research*, vol. vol. 148, pp. 14-27, 2003.
- [20] B. Roy, "Méthologie Multicritère d'Aide à la Décision," *Economica, Paris*, 1985.
- [21] C. Bana, *Readings in multiple criteria decision aid*. Berlin: Springer-Verlang, 1990.
- [22] P. Vincke, *Basic Concepts of Preference Modelling. In C. A. Bana e Costa (Ed.), Readings in Mutiple Criteria Decision Aid*. Berlin: Springer Verlag, 1990.
- [23] D. Bouyssou, *Building Criteria: A Prerequisite for MCDA. In C. A. Bana e Costa (Ed.)*. Berlin: Springer Verlag, 1990.
- [24] K. L. Boettcher and A. H. Levis, "Modelling the interacting decision maker with bounded rationality," *LIDS-P*, p. 1110, 1981.

- [25] B. Roy and D. Bouyssou, "Decision Aid: An elementary introduction with emphasis on multiple criteria," *Investigación Operativa*, vol. 2, pp. 95-110, 1991.
- [26] E. Georgopoulou, *et al.*, "Design and implementation of a group DSS for sustaining renewable energies exploitation," *European J. of Operational Research*, vol. 109, pp. 483-500, 1998.
- [27] M. Rogers and M. Bruen, "Using ELECTRE to an option choice problem within an environmental appraisal - Three case studies from the Republic of Ireland," presented at the International Conference on methods and applications of Multicriteria Decision Making, Mons, Belgium, 1997.
- [28] T. Spengler, *et al.*, "Development of a multiple criteria based decision support system for environmental assessment of recycling measures in the iron and steel making industry," *Journal of Cleaner Production*, vol. 6, pp. 37-52, 1998.
- [29] DETR, "Multi-Criteria Analysis. A Manual," T. a. t. R. Department of the Environment, Ed., ed. London, 2000.
- [30] M. Perlot, "A common framework for describing some outranking methods," *Journal of Multicriteria Decision Analysis*, vol. 6, pp. 86-92, 1997.
- [31] B. Roy, *The outranking approach and the foundations of ELECTRE methods. Readings in multiple criteria decision aid*, . Berlin: Springer Verlag, 1991.
- [32] J. C. Pomerol and S. Barba-Romero, "Multi-criterion decisions in management: Principles and practice," *Kluwer, Massachusetts, USA.*, 2000.
- [33] P. C. Fishburn, "Non-compensatory preferences," *Synthese*, vol. 33, pp. 393-403, 1976.
- [34] B. Roy, "Classement et choix en présence de point de vue multiples: Le méthode ELECTRE," *Revue Francaise d'Informatique et de Recherche Opérationnelle*, vol. 8, pp. 57-75, 1968.
- [35] J. P. Brans, *et al.*, "How to select and how to rank projects: The PROMETHEE method.," *European Journal of Operational Research*, vol. 24, pp. 228-238, 1986.

- [36] M. Roubens, "Preference relation in actions and criteria in multicriteria decision making," *European Journal of Operational Research*, vol. 10, pp. 51-55, 1982.
- [37] J. C. Vansnick, "On the problem of weights in multiple criteria decision making, the non-compensatory approach," *European Journal of Operational Research*, vol. 24, pp. 288-294, 1986.
- [38] H. Voogd, "Multicriteria evaluation for urban and regional planning," presented at the Pion, London, 1983.
- [39] E. Hinloopen, *et al.*, *The REGIME method: a new multicriteria technique.* "Essays and surveys on multiple criteria decision making" vol. 13: Springer, 1983.
- [40] B. Matarazzo, "Multicriterion analysis of preferences by means of pairwise actions and criterion comparisons," *Applied Mathematics and Computations*, vol. 18, pp. 119-141, 1986.
- [41] J. L. Orner and C. W. Kirkwood, "Decision analysis applications in the operations research literature 1970-1989," *Operations Research*, vol. 39, pp. 206-219, 1991.
- [42] V. Belton and S. Theodor, *Multiple Criteria Decision Analysis: An integrated approach*: Kluwer Academic Publishers, 2003.
- [43] P. Hanse, *et al.*, *quasi-kernels of outranking relations* vol. 30, 1976.
- [44] B. Roy and P. Bertier, *La méthode Electre II*: OR '72, North-Holland Publishing Company, 1973.
- [45] D. Bouyssou and M. Pirlot, "A general framework for the aggregation of semiorders," *Tech. rep., ESSEC*, 1997.
- [46] B. Roy, "un algorithme de classement fondé sur une représentation floue des préférences en présence de critères multiples," *Cahiers du CERO*, vol. 20, pp. 3-24, 1978.
- [47] B. Roy and J.-C. Hugonnard, "Ranking of suburban line extension projects on the Paris metro system by a multicriteria method," *Transportation Research*, vol. 16A, pp. 301-312, 1982.

[48] B. Roy and D. Bouyssou, "Aide Multicritère à la Décision: Méthodes et cas," *Economica, Paris*, 1993.

[49] P. Perny, "Multicriteria filtering methods based on concordance and non-discordance principles " *Annals of Operations Research*, vol. 80, pp. 137-165, 1998.

[50] A. D. R. Goicoechea, *et al.*, *Multiobjective Decision Analysis with Engineering and Business Applications*. New-York: Wiley, 1982.

[51] J. G. Siskos, *et al.*, "Title," unpublished|.

[52] D. Bouyssou, *On some properties of outranking relations based on a concordance-discordance principle*: Springer, 1992.

[53] A. Ostanello, *Outranking relations*. Berlin: Springer Verlag, 1985.

[54] P. Vincke, *The outranking Approach*. Dordrecht: Kluwer Academic, 1999.

[55] D. Bouyssou, "Outranking relations: do they have special preproperties?," *Journal of Multicriteria Decision Analysis*, vol. 5, pp. 99-111, 1996.

[56] P. Perny, "Modélisation, agrégation et exploitation des préférences floues dans une problématique de rangement: Bases axiomatique, procédures et logiciels," Université de Paris-Dauphine, 1992.

[57] J. P. Brans and P. Vincke, "A preference ranking organization method," *Management Science*, vol. 31, pp. 647-656, 1985.

[58] P. Perny and B. Roy, *The use of fuzzy Outranking Relations in Preference Modelling* vol. 49, 1992.

[59] N. Belacel, "Multi-criteria assignment method PROAFTN: Methodology and medical application," *European Journal of Operational Research*, vol. 125, pp. 175-183, 2000.

[60] L. Henriët, "Systèmes d'évaluation et de classification multicritère pour l'aide à la décision," Thèses de Doctorat, Université Paris Dauphine, ,France, 2000.

[61] "Plan Nacional Integrado de Residuos 2008-2015 (PNIR)," ed. Madrid, 2008.

[62] J. Pijuan, Valls, A., Passuello, A., Schuhmacher, M., , "Evaluating the impact of sewage sludge application on agricultural soils," presented at the XV Congreso español sobre tecnologías y lógica fuzzy (ESTYLF), Huelva (Spain), February 2010.

[63] A. Valls, Schuhmacher, M., Pijuan, J., Passuello, A., Nadal, M., Sierra, J., , "Preference assessment for the management of sewage sludge application on agricultural soils," *International Journal of Multicriteria Decision Making*, vol. Vol. 1, pp. 4-24, 2010.

[64] A. Jessop, "Sensitivity and robustness in selection problems," *Computers & Operations Research*, vol. 21, pp. 607-622, 2004.

Annex