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Vorticity and curvature at a stream surface

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Abstract

If S is a stream surface in a flow, we show the relationship between the three components of the vorticity field on S and the curvatures of the streamlines (geodesic torsion, normal curvature and geodesic curvature).

MSC: 76A02; 53Z05

Keywords: Vorticity; Stream surfaces; Curvature

1. Introduction

The study of the vorticity on the interfacial surfaces is of considerable interest. Recent works relate the geometry of the surfaces with the components of the vorticity, for example Wu [1], and Dopazo, Lozano and Barreras [2]. If S is a steady and free surface (both tangential components of the stress tensor vanish at the surface), Longuet-Higgins [3] shows the relationship between the tangential components of the vorticity field on S and the normal curvatures of S. In this paper we consider the case where S is a stream surface, and we find the relationship between the three components of the vorticity field on S and the curvatures of the streamlines.

We consider a stream surface S in a flow; that is: let S be a smooth surface of $\mathbb{R}^3$ (oriented Euclidean three-dimensional space) tangent to the smooth velocity vector field $\vec{u} = \vec{u}(p,t)$ of the flow at any fixed time t, which moves with time. Let $\vec{x}$ be a parametrization of the smooth surface S:

$$\begin{aligned} &\vec{x} : U \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^3, \\ &\quad (\xi_1, \xi_2) \longrightarrow \vec{x}(\xi_1, \xi_2) = \big(x(\xi_1, \xi_2), y(\xi_1, \xi_2), z(\xi_1, \xi_2)\big). \end{aligned} \tag{1}$$

Let $\vec{N}(\xi_1, \xi_2)$ be the normal vector given by the parametrization

$$\vec{N} = \frac{\partial\vec{x}/\partial\xi_1 \times \partial\vec{x}/\partial\xi_2}{\|\partial\vec{x}/\partial\xi_1 \times \partial\vec{x}/\partial\xi_2\|}.$$

We consider the velocity vector field $\vec{u} = \vec{u}(p,t)$ and the vorticity vector field $\vec{\omega}(p,t) = \mathrm{curl}(\vec{u})$ on the stream surface S. Let $\vec{u}^{\perp}$ be the tangent vector field on S such that $\{\frac{\vec{u}}{q}, \frac{\vec{u}^{\perp}}{q}, \vec{N}\}$ is an orthonormal, direct basis (positively

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