Simplified Procedure for the Design and Optimisation of Oval Tensile Spoke Wheels for Roof Structures

Rodrigo Martín-Sáiz, Dr, Escola Tècnica Superior d'Arquitectura, Universitat Rovira i Virgili, Reus, Spain; **Blas Herrera**, Prof., Departament d'Enginyeria Informàtica I Matemàtiques, Universitat Rovira I Virgili, Tarragona, Spain.
Contact: rodrigo.martin@urv.cat
DOI: 10.1080/10168664.2024.2383582

Abstract

This paper presents a simplified procedure for the design and shape optimisation of oval tensile spoke wheels to be used in roof structures for grandstands in sports stadiums. This procedure combines graphical methods with simple numerical methods to obtain the geometrical definition of an oval wheel and to estimate the prestressing forces. It also allows for the compression forces in the outer ring to be estimated and compared with those in a circular wheel. This makes it easy to make small geometrical adjustments for optimisation. Application examples are given for each step of the procedure. Several wheel designs are defined, showing how the shape of the inscribing oval determines the magnitude of the forces in the outer ring.

Keywords: ovals; superellipses; tensile spoke wheels; graphic statics; roof structures.

Introduction

Ovals

Three ovals with two perpendicular axes of symmetry, formed by arcs of tangent circles, are considered (*Fig. 1*). The simplest ovals of this type have four centres and four arcs with two different radii; they are commonly known as four-centered ovals and also as quadrarcs. The two centres of the arcs with the smaller radius lie on the x-axis line of the major axis, and the two centres of the arcs with the larger radius lie on the y-axis line of the minor axis. The next most complex ovals of this type have eight centres and eight arcs of three different radii, and are commonly known as eight-centered ovals. The geometric quality of the approximation of a given ellipse is higher using eight-centered ovals (with the arcs ordered from smallest to largest) than using four-centered ovals. A general and thorough discussion of this fact can be found in Ref. [1]. With the arcs ordered from smallest to largest, the two centres of the arcs with the smaller radius lie on the x-axis line of the major axis, the two centres of the arcs with the larger radius lie on the y-axis line of the minor axis, and the four centres of the arcs with the intermediate radius lie outside of the two symmetry axes. On the other hand, if we take an oval with eight centres and place the four arcs with the smallest radius between the other arcs with the largest radius, we obtain a shape similar to a superellipse[2]. Therefore, in this oval, the two centres of the arcs with the larger radius lie on the y-axis line of the minor axis, the two centres of the arcs with the intermediate radius lie on the x-axis line of the major axis, and the four centres of the arcs with the smaller radius lie outside the symmetry axes.

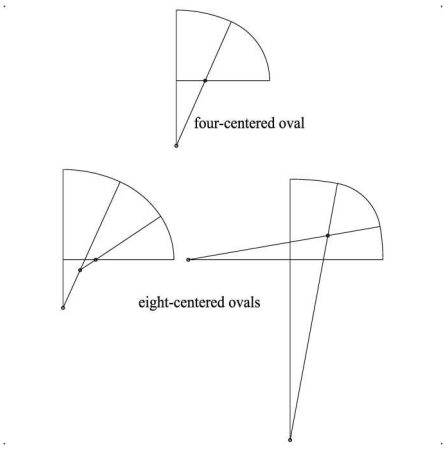

Fig. 1: First quadrant of three ovals with two perpendicular symmetry axes

The first quadrants of four-centered ovals are defined by Eq. (1), while the first quadrants of eight-centered ovals are defined by Eq. (2). In these equations, the parameters h_i, k_i are the coordinates (h_i, k_i) of the centres, r_i are the radii of each of the circle arcs and t_i are the abscissae of the tangency points, all in clockwise order starting from the arc closest to the y-axis.

$$y = \begin{cases} k_1 + \sqrt{r_1^2 - x^2}; 0 \le x < t_1 \\ \sqrt{r_2^2 - (x - h_2)^2}; t_1 \le x \le a \end{cases} \quad (1)$$

$$y = \begin{cases} k_1 + \sqrt{r_1^2 - x^2}; 0 \le x < t_1 \\ k_2 + \sqrt{r_2^2 - (x - h_2)^2}; t_1 \le x < t_2 \\ \sqrt{r_3^2 - (x - h_3)^2}; t_2 \le x \le a \end{cases} \quad (2)$$

The Shape of Sports Stadiums

The outer perimeter of the stands in many sports stadiums is designed as a four-centered oval or an eight-centered oval. Four-centered oval shapes are typical for athletics arenas, for example the Olympic stadiums built for Rome 1960, Montreal 1976, Los Angeles 1984, Seoul 1988 and Athens 2004[3]. Four-centered ovals can also be found in football stadiums, such as Maracana in Rio de Janeiro[4] or FC Krasnodar stadium in Russia[5]. Eight-centered ovals similar to ellipses are also typical for athletics arenas, for example the Olympic stadiums built for Berlin 1936 and London 2012[3], or the Slaski Stadium in Silesia[6]. Eight-centered ovals similar to super-ellipses are typical for sports stadiums with a rectangular playing field, such as football, rugby or American football stadiums such as the Georgia Dome in Atlanta[7], the Allianz Arena in Munich[8], the Commerzbank Arena in Frankfurt[9], or the Arena da Amazonia in Manaus[10]. *Table 1* shows nine of the above-mentioned oval stadiums, sorted according to the type of oval.

Tensile Spoke Wheels

The tensile spoke wheel (TSW) is probably the most widely used type of structure for grandstand roofs in