



## Original article

## Systematic calculation of flying buttress parameters by means of geometric regression

Albert Samper<sup>a,1</sup>, Blas Herrera<sup>b,2,\*</sup>, Agustí Costa-Jover<sup>c,3</sup><sup>a</sup> Universitat Rovira i Virgili and IRH-UDG, Escola Tècnica Superior d'Arquitectura, Reus, Spain<sup>b</sup> Universitat Rovira i Virgili, Departament d'Enginyeria Informàtica i Matemàtiques, Tarragona, Spain<sup>c</sup> Universitat Rovira i Virgili, Escola Tècnica Superior d'Arquitectura, Reus, Spain

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## ABSTRACT

This paper investigates the geometric parameters of a flying buttress: position of the center  $\mathcal{O}$  of the intrados arch, radius  $\mathcal{R}$  of the intrados arch, inclination  $\alpha$ , rise  $\mathcal{F}$ , span distance  $\mathcal{L}$  and horizontal thickness  $\mathcal{E}$  of the culée. Using photogrammetrical techniques, point cloud control, vector redrawing and geometric regression, this paper provides an objective, non-arbitrary procedure to determine the center  $\mathcal{O}$  and the radius  $\mathcal{R}$  of a flying buttress arch. This results in two outcomes: 1) Non-arbitrary determination of the remaining parameters  $\alpha$ ,  $\mathcal{F}$ ,  $\mathcal{L}$  and  $\mathcal{E}$ ; 2) Non-arbitrary classification of flying buttress arches into two types, according to the criterion laid down by Viollet-le-Duc in 1854.

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## 1. Introduction

The construction of churches and Gothic cathedrals is a complex issue which has drawn the attention of many researchers in recent history. There are scarcely any historical records from the Gothic period which address design issues, and the few existing documents in this regard date from the late Gothic period [1]. Several investigations on the history of construction were conducted in the mid-nineteenth century, such as the papers by Robert Willis [2] (Willis, 1842) and Auguste Choisy [3], among many others, but Viollet-le-Duc [4] was the first researcher who presented a complete theory on Gothic construction. This paper intends to further knowledge in this area, focusing on one of the most representative elements of Gothic architecture: the flying buttress.

Recent monographic investigations on flying buttresses are extremely scarce. The papers written by Eugène Lefèvre-Pontalis [5], Jacques Heyman [6] and Robert Mark [7] –the two latter focusing on the overall stability of buildings– were the precursors to research on these architectural elements. John Fitchen also looked at the construction of Gothic cathedrals and specifically analyzed the behavior of flying buttresses [8,9]. Other more recent investigations,

such as [10] and [1, 11], also furthered the structural analysis of these buildings and examined their design and proportions.

Recent investigations specifically dealing with flying buttresses include several papers focused on structural matters, such as [12] and [13], who provided a method to analyze flying buttresses based on Heyman's approach. The authors of reference [14] developed a method of analysis based on compression lines, complementing their structural analysis with a study of the geometric layout of the intrados arch. Later, [15] examined flying buttresses from a purely geometric point of view, and provided a method to study arches having two centers.

A flying buttress is determined by the following geometric parameters: position of the center  $\mathcal{O}$  of the intrados arch, radius  $\mathcal{R}$  of the intrados arch, inclination  $\alpha$ , rise  $\mathcal{F}$ , span distance  $\mathcal{L}$  and horizontal thickness  $\mathcal{E}$  of the culée. These parameters can be seen in Fig. 1. From a geometric point of view, we will show that all these parameters are determined by the center  $\mathcal{O}$  and the radius  $\mathcal{R}$  of the circle which defines the intrados arch of the flying buttress. In theory, it would be enough to choose three random points from the edge contour of the intrados arch and then draw the perpendicular bisectors of the resulting segments in order to determine  $\mathcal{O}$  and  $\mathcal{R}$ . However, this method is not so evident or valid, as will be made clear later in this paper.

The lack of accuracy during the construction process, the deterioration due to the passing of time, seismic disturbances, structural deformations and other incidents of diverse nature may result in a change of the geometric type of the arch which was originally designed, and may also cause that the arch's center  $\mathcal{O}$  and the arch radius  $\mathcal{R}$  vary depending on the three points which are chosen.

\* Corresponding authors.

E-mail addresses: [albert.samper@urv.cat](mailto:albert.samper@urv.cat) (A. Samper), [blas.herrera@urv.cat](mailto:blas.herrera@urv.cat) (B. Herrera), [agusti.costa@urv.cat](mailto:agusti.costa@urv.cat) (A. Costa-Jover).<sup>1</sup> Orcid: 0000-0002-4795-2127<sup>2</sup> Orcid: 0000-0003-2924-9195.<sup>3</sup> Orcid: 0000-0002-6194-3243