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An Analogue of Thébault's Theorem Linking the Radical Center of Four Spheres with the Insphere and the Monge Point of a Tetrahedron

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ABSTRACT

In 1953, Victor Thébault conjectured a link between the altitudes of a tetrahedron and the radical center of the four spheres with the centers at the vertices of this tetrahedron and the corresponding tetrahedron altitudes as radii. This conjecture was proved in 2015. In this paper, we propose an analogue of Thébault's theorem. We establish a link between the radical center of the four spheres, the insphere, and the Monge point of a tetrahedron.

Keywords: *Solid geometry, Thébault's theorem, tetrahedron, radical center of spheres, Monge point;, insphere.*
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1. Introduction

The famous French problemist Victor Thébault (1882 – 1960) conjectured the following geometric fact linking the radical center of four spheres with other elements of a tetrahedron. Let AA', BB', CC', DD' be the altitudes of a tetrahedron $ABCD$ with feet A', B', C', and D', respectively. Let P be the radical center of the spheres with centers A, B, C, D and radii AA', BB', CC', and DD', respectively. Then each plane passing through the midpoint of the segment $B'C'$, $C'A'$, $A'B'$, $D'A'$, $D'B'$ or $D'C'$ perpendicular to the segment BC, CA, AB, DA, DB or DC, respectively, contains the point P [5, 6]. This hypothesis was proved in 2015 in [3]. We found a fact that is similar to the result of Thébaultt, but now linking the radical center of four spheres with the insphere and the Monge point of a tetrahedron. To avoid ambiguities, we give here the following definitions.

Let T be a tetrahedron in the Euclidean space $\mathbb{E}^3$. A sphere that touches four faces of the tetrahedron T is called the *insphere* of T. There are six planes, each of which passes through the midpoint of the edge of the tetrahedron T perpendicular to its opposite edge. These six planes have a common point, which is called the *Monge point* of T [4, 7, 8].

The *power* of a point P with respect to a sphere ω with a center O and radius R is the number

$$Pow(\omega, P) = |OP|^2 - R^2.$$

Let ω_1, ω_2, ω_3, and ω_4 be spheres with noncomplanar centers in $\mathbb{E}^3$. There exists an unique point that has the same power with respect to each of these spheres. This point is called the *radical center* of the spheres ω_1, ω_2, ω_3, and ω_4 [2, 1].

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