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Two Conjectures of Victor Thébault Linking Tetrahedra with Quadrics

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Abstract. We prove two of Thébault's conjectures. The first (1949) links four lines, that they are rulings of hyperbolic paraboloids or that they are coplanar, with orthocentric or isodynamic tetrahedra, respectively. The second (1953) links the radical center of four spheres with elements of tetrahedra.

1. Introduction

It is very well known that an Euclidean tetrahedron $T \equiv ABCD \subset \mathbb{A}^3$, where $\mathbb{A}^3$ is the Euclidean affine space, is called *orthocentric*, by definition, if the lines through the vertices which are orthogonal to the opposite faces are concurrent; and T is called *isodynamic*, by definition, if the segments that join the vertices with the incenter of the opposite faces are concurrent. It is also very well known that the radical center of four spheres is a point P such that the four powers of P with respect to the four spheres are are equal.

In [12] the famous French problemist Victor Thébault (1882-1960) conjectured the following: In a tetrahedron $T \equiv ABCD$, the planes tangent at A, B, C, D to the circumsphere of T cut the planes of the opposite faces in four lines. A necessary and sufficient condition for these four lines to be rulings of a hyperbolic paraboloid is that T be orthocentric, and a necessary and sufficient condition for these four lines to be coplanar is that T be isodynamic.

But the above conjecture, since 1949 has remained open.

Also, in [13] Victor Thébault conjectured the following: In a tetrahedron $ABCD$, let A', B', C', D' be the feet of the altitudes AA', BB', CC', DD'. The planes drawn through the midpoints of $B'C'$, $C'A'$, $A'B'$, $D'A'$, $D'B'$, $D'C'$ perpendicular to BC, CA, AB, DA, DB, DC respectively, are concurrent at a point P, which is the radical center of the spheres described with the vertices A, B, C, D as centers and with the altitudes AA', BB', CC', DD' as radii.

This conjecture, since 1953 has remained open.

In this paper we prove affirmatively these two results; we will call them theorems.

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