On the Radical Center of Four Spheres*

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We provide two families of vector formulas that determine the radical center of four spheres. As some applications, we show formulas for points in similar situations to those of the Monge point, provide new geometric results for radical centers, and we give a new proof of a conjecture by Victor Thébault from 1953.

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Introduction

In this article, we provide two families of vector formulas that determine the radical center of four spheres. As some applications, we show formulas for points in similar situations to those of the Monge point, provide new geometric results for radical centers, and we give a new proof of a conjecture by Victor Thébault from 1953 [1, 2].

Throughout this article, the authors use the following terms and notations. The Euclidean vector connecting an initial point P with a terminal point Q (in the three-dimensional Euclidean affine space $\mathbb{E}^3$) is denoted by $\overrightarrow{PQ} = -P + Q$. Zero vector in $\mathbb{E}^3$ is denoted by $\overrightarrow{0}$. The notation $\overrightarrow{AB} \cdot \overrightarrow{CD}$ denotes the dot product of two Euclidean vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$. The notation $\overrightarrow{AB} \times \overrightarrow{CD}$ denotes the cross product of two Euclidean vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

To avoid ambiguities, the following definitions are given: Let $\mathcal{T}$ be a tetrahedron in the Euclidean space $\mathbb{E}^3$. A sphere that touches four faces of the tetrahedron $\mathcal{T}$ is called the *insphere* of $\mathcal{T}$. Every tetrahedron has a special point known as the *Monge point*, which is the intersection of the six planes that are perpendicular to a given edge and pass through the midpoint of the opposite edge.

*Vol.28, No.8, DOI: https://doi.org/10.1007/s12045-023-1661-x