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ORIGINAL PAPER

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Design of bending-moment-free catenary arches of finite thickness: a discrete geometric approach

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Abstract Although a one-dimensional catenary arch subjected to its self-weight is simply compressed and bending-moment-free, when this arch is three-dimensional and has a constant rectangular cross-section of finite dimensions, the centroids of its voussoirs are displaced towards the extrados. This displacement prevents the arch from being geometrically compatible with a bending-moment-free state, a contradiction known as the catenary arch bending-moment-free paradox [1]. Firstly, this paper demonstrates that this paradox is not exclusive to rectangular cross-sections, but extends to any trapezoidal cross-section, ruling out any continuous solution to the centroid eccentricity problem. This impossibility result motivates a discrete approach to solve a geometrical problem: find the voussoir geometry such that the eccentricity between the centroidal axis and the catenary curve is zero for all voussoirs, subject to the constraint of uniform volume distribution along the catenary axis. The proposed method is based on the discretisation of a catenary curve into small segments of equal length, such that both the volumetric centroids of the voussoirs and the centroids of the normal sections are located on the catenary curve. Three alternative designs for the voussoir shape are developed and analysed. The method is founded on a closed-form analytical geometric formulation, and the accuracy of the solution increases with the number of voussoirs. Furthermore, a closed-form expression for the minimum number of voussoirs required to achieve a prescribed accuracy is derived. The outcomes obtained from the design of various arches reveal the geometric and structural properties of the resulting forms. The proposed design has been verified by graphic statics, confirming that the resulting arches are geometrically compatible with a bending-moment-free state. The described method could be part of an integrated process of designing and manufacturing voussoirs using 3D printing for more efficient arches and vaulted structures.

1 Introduction

1.1 Catenary as the perfect arch

According to the principle of inversion formulated by Robert Hooke in 1675 [2], a catenary arch should be simply compressed and bending-moment-free, just as its inverse form, a hanging chain, is simply tensioned and bending-moment-free. A chain can be regarded as a one-dimensional element because the dimensions of its cross-section can be neglected, being negligible compared to its length. Consequently, a hanging chain forms a curved axis along which its self-weight is evenly distributed. This curved axis is a simply tensioned

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